A quadrivalent approach to anaphora

Patrick D. Elliott (joint work with Amir Anvari)*
Heinrich-Heine University Düsseldorf

October 24, 2025

- Handout: `https://patrickdelliott.com/pdf/dip.pdf`
- Contact: `patrick.d.elliott@gmail.com`

Contents

1 Background: dynamic semantics and anaphoric accessibility

- Achievements of 'first-generation' Dynamic Semantics (DS): truth-conditionally adequate semantics for discourse and donkey anaphora (Heim 1982, Groenendijk & Stokhof 1991, etc.):

*A little history: Amir and I joined forces only after realizing that we had been independently developing the same theory for some time. Any mistakes or inaccuracies in this version are most likely due to my own idiosyncratic take on this material. An 'official' version can be found here: `https://ling.auf.net/lingbuzz/009360`.

(1) A^x linguist walked in, and she$_x$ immediately sat down. *discourse anaphora*

(2) If a^x linguist walked in, she$_x$ immediately sat down. *donkey anaphora*

- Accounts for a broad set of 'accessibility' facts (see esp. Groenendijk & Stokhof 1991). E.g., negation renders an indefinite in its scope 'inaccessible' to a subsequent pronoun.

(3) ✗I haven't read a^x paper by Montague; It$_x$'s too difficult for me to understand.

- Disjunction also renders an indefinite in its scope inaccessible to a subsequent pronoun:

(4) ✗Either Juli has a^x bicycle, or she drives everywhere. It$_x$'s parked outside.

- Furthermore, anaphoric-dependencies not possible *between* disjuncts.

(5) ✗Either Juli has a^x bicycle, or it$_x$'s parked outside.

- There are problems however, as already recognized by, e.g., Heim 1982, Groenendijk & Stokhof 1991, centered around the treatment of negation and disjunction.
- First-generation DS invalidates DNE, but this does not reflect how anaphora works in natural language (Groenendijk & Stokhof 1991: §5.1):[1]

(6) It's not true that Juli doesn't have a sibling^x.
She brought him$_x$ to the party last Saturday.
(cf. *Juli isn't an only child. ✗She brought him to the party last Saturday.*)[2]

- Another problem: anaphoric dependencies are possible between disjuncts under certain conditions (Evans 1977).

(7) Either there isn't a^x bathroom in this house, or it$_x$'s in a funny place.
Attributed to Barbara Partee

(8) Either Mary doesn't have a^x sibling, or he$_x$ also has red hair.
(Cf. ✗ *Either Mary isn't an only child, or he also has red hair*)

- Strikingly, unlike other cases of discourse/donkey anaphora, where accessibility is strictly asymmetric, bathroom disjunctions permit a cataphoric dependency[3]

(9) Either it$_x$'s in a funny place, or there isn't a^x bathroom in this house.

(10) Either he$_x$ also has red hair, or Mary doesn't have a^x sibling.
(cf. ✗ *Either he also has red hair, or Mary is an only child*)

- Another problem associated with disjunction: indefinites become accessible, if there is a 'parallel' indefinite in each disjunct (Groenendijk & Stokhof 1991, Stone 1992).[4]

---

[1] Gotham (2019) makes the case for double-negation giving rise to an additional uniqueness inference, but Elliott (2020, 2023), Mandelkern (2022) show that this is easily cancellable. Consider a variation of Heim's (1982) sage plant sentences: "It's not the case that Sally DIDN'T buy a^x sage plant; she bought eight others along with it$_x$!".

[2] Controls exploit a contextually equivalent sentence without an indefinite; designed to show that antecedent accommodation is not responsible for acceptability.

[3] Chierchia (1995) suggests that donkey cataphora is sometimes available in conditional sentences, but the relevant examples are marginal.

[4] Groenendijk & Stokhof (1991) do suggest a way of accounting for (11), but doing so sacrifices an account of (4), and in any case doesn't shed any light on (7).

(11) If a^x professor or an^x assistant professor attends the meeting of the university board, then he$_x$ reports to the faculty. (Groenendijk & Stokhof 1991: example 13)

- These problems (especially DNE and bathroom disjunctions), have been discussed extensively in the literature. See, e.g., Krahmer & Muskens (1995) for an influential account of DNE and bathroom disjunctions that exploits bilaterality, and Gotham (2019), Hofmann (2022, 2025), Aloni (2023) for important subsequent work from a dynamic perspective.
- These accounts offer valuable insights, but do not establish a clear connection between anaphoric accessibility and presupposition projection.[5]
- Much like anaphora, presupposition projection exhibits asymmetries in conjunctive sentences (Karttunen 1973; see Mandelkern et al. 2020 for experimental confirmation).
  - (12b) but not (12a) presupposes that *Mary used to do yoga* (examples from Mandelkern et al. 2020: p. 477).

(12) *I have no idea if Mary has ever done yoga, but...*
  a. If [Mary used to do Jivamukti yoga and she stopped doing yoga], then Matthew will interview her for this story.
  b. (!!!) If [Mary stopped doing yoga, and she used to do Jivamukti yoga], then Matthew will interview her for this story.

- Presupposition projection in disjunctions in *symmetric*; negation implicated much as with bathroom sentences (Karttunen 1973; Kalomoiros & Schwarz 2024 for experimental confirmation).

(13) *I have no idea if Mary has ever done yoga, but...*
  a. If [either Mary has never done yoga, or she stopped doing yoga], Matthew won't interview her for this story.
  b. If [either Mary stopped doing yoga, or she has never done yoga], Matthew won't interview her for this story.

- Some recent accounts of anaphora forge a tight connection with presupposition projection: specifically, Rothschild 2017, Elliott 2020, 2023, Mandelkern 2022, Spector 2024, Heim 2024, but none (with the possible exception of Chatain 2025[6]) are able to satisfactorily account for the full range of facts.
- A brief note on my own previous work: In Elliott (2020, 2023), I developed a trivalent dynamic semantics, building on insights from Groenendijk & Stokhof (1991), Krahmer & Muskens (1995), van den Berg (1996), Charlow (2020). The logical connectives are defined as a dynamic lifting of the Strong Kleene connectives, but the lifting procedure itself introduces a left-to-right bias which makes cataphora difficult to capture without stipulation.
- Rothschild (2017), Heim (2024) manage to successfully account for the asymmetric availability of discourse anaphora by exploiting a trivalent theory of presupposition projection—specifically, Peters's (1979) trivalent conjunction.

[5] Heim (1982, 1983) of course famously develops a unified account of anaphora and presupposition from a dynamic perspective. The dynamic account of presupposition has however fallen from favor given the availability of more explanatory/predictive alternatives in, e.g., the trivalent tradition (see, e.g., Beaver & Krahmer 2001.)

[6] Chatain's interesting account requires abandoning a treatment of pronouns as variables, favoring instead an E-type analysis. This requires positing additional mechanisms in order to explain how pronouns inherit their descriptive content from an indefinite antecedent.

- We find this approach very promising, but their accounts fail to capture DNE or bathroom disjunctions (as discussed in the following section).

- Mandelkern (2022) accounts for discourse anaphora by exploiting Schlenkerian local contexts (Schlenker 2009, 2010), and Spector (2024) via transparency (Schlenker 2008).
  - Bathroom anaphora is accounted for, but neither theory as stated accounts for bathroom cataphora while blocking cataphora in conjunctions; Schlenker's theories of presupposition are either strictly symmetric, or strictly asymmetric.

- In the following, we build primarily upon Rothschild (2017), Heim (2024), integrating anaphora into the trivalent tradition: variable (a-)symmetries in presupposition filtering are captured by adopting the truth-tables in Table 1 (a hybrid of Peters 1979 and Strong Kleene, derived from George's 'disappointment' algorithm).

$\phi \wedge^{mk} \psi$	1	0	#
1	1	0	#
0	0	0	0
#	#	#	#

$\phi \vee^{sk} \psi$	1	0	#
1	1	1	1
0	1	0	#
#	1	#	#

Figure 1: Middle Kleene conjunction + Strong Kleene disjunction

- This raises an explanatory challenge: how are the truth-tables we assume derived? We address this, but not until §8.

- For now, the challenge is simply to develop a descriptively adequate theory of anaphora, grounded in the trivalent tradition. Doing so will require introducing a non-# failure value.

2 Assertive and presuppositional analyses of indefinites

- In order to capture the correspondence between presupposition projection, and anaphoric accessibility, it will be important to have pronouns *qua* variables to introduce presuppositions. This is accomplished by assuming partiality of assignments.

- In the trivalent tradition, # models presupposition failure (Peters 1979, Beaver & Krahmer 2001, George 2008a,b, etc.).

(14) **Atomic sentences:**

$$[\![P(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g), g(x) \in P_w \\ 0 & x \in dom(g), g(x) \notin P_w \\ \# & \text{otherwise} \end{cases}$$

- Before developing the semantics further, let's consider what it means for a presupposition about a variable to be satisfied in a discourse context.

- We assume a standard dynamic notion of the discourse context as a set of world-assignments pairs (a *file*; Heim 1982).

- Assertion of a sentence ϕ relative to such a discourse context is subject to a requirement that any presuppositions are contextually entailed Stalnaker 1976, von Fintel 2008.
  - The bridge principle in (55b), in concert with (14), means that assertion of $P(x)$ requires every $(w, g) \in c$ to be one at which x is defined (i.e., Heimian *familiarity*).

(15) **Stalnaker's bridge:** after von Fintel 2008

Assertion of a sentence ϕ at a discourse context c is undefined if $\exists(w, g) \in c$, $[\![\phi]\!]^{w,g} = \#$.

- We'll return to how context update works momentarily, but first: moving on to filtering and accessibility. Our initial goal in light of the data in the previous section:
  - in a sentence of the form $\exists_x P(x) \wedge Q(x)$ the presupposition of $Q(x)$ is *filtered.*
  - In a sentence of the form $Q(x) \wedge \exists_x P(x)$ the presupposition of $Q(x)$ *projects.*
- In order to model asymmetries in conjunctive sentences, we'll assume that conjunction has a *Middle Kleene* semantics (Peters 1979), following Rothschild (2017), Heim (2024).

$\phi \wedge^{mk} \psi$	1	0	#
1	1	0	#
0	0	0	0
#	#	#	#

Figure 2: Middle Kleene conjunction

- In light of Figure 2, the presupposition of an initial conjunct always projects, but the presupposition of a subsequent conjunct may be filtered.
- Since $Q(x)$ *presupposes* that x is defined, then an perhaps an existential statement should *assert* that this is so.
  - We call this the 'assertive analysis'; it's based on the accounts of Rothschild 2017, Heim 2024.[7]

(16) **Existential statements (assertive analysis):** after Rothschild 2017, Heim 2024

$$[\![\exists_x^A P(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g), g(x) \in P_w \\ 0 & \text{otherwise} \end{cases}$$

- Immediate problem for the assertive analysis: negated existential statements are too weak (assuming classical negation).[8]
  - The statement $\neg\exists_x^A P(x)$ is true at g if $x \notin dom(g)$, even if $P \neq \emptyset$.
- One way of side-stepping this: drop the bivalence assumption; trivalence allows one to keep the falsity clause appropriately strong.

(17) **Existential statements (presupp. analysis):** after Mandelkern 2022, Spector 2024

$$[\![\exists_x^P P(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g), g(x) \in P_w \\ 0 & P_w = \emptyset \\ \# & \text{otherwise} \end{cases}$$

- $\exists_x^P P(x)$ thereby presupposes that *if there's a P, then x is a P.*
  - Mandelkern (2022) calls this the *Witness Presupposition.*

[7]For ease of exposition, I focus here on $\exists_x \phi$ where ϕ is atomic. It should be easy to reconstruct the more general definition.

[8]Rothschild 2017, Heim 2024 propose to solve this by introducing a closure operator, which leads to additional complications, particularly in the analysis of bathroom sentences.

- This clearly delivers an appropriate truth clause for negated existential statements, assuming a standard trivalent semantics for negation.
- A problem for the presuppositional analysis:
  - $\exists_x^P P(x)$ is # at any (w, g) such that $P \neq \emptyset$ and either $x \notin dom(g)$ or $g(x) \notin P_w$.
  - A standard bridge principle therefore predicts that an existential statement should only be assertable if x is already defined at all the P-worlds.
  - However, existential statements are usable even if x has not been introduced as a discourse referent (to use a familiar dynamic idiom).
- Mandelkern 2022 and Spector 2024 solve this problem in distinct but related ways; Mandelkern (2022) sidesteps this problem by introducing an additional dimension of meaning, leaning on Schlenkerian local contexts (Schlenker 2009, 2010).[9]
- Spector 2024 abandons an interpretation of # as presupposition failure, incorporating Schlenker's transparency theory over and above trivalence.
- Our goal is to maintain an interpretation of # as presupposition failure—we maintain that both the assertive and presuppositional analyses are, in different ways, correct.
  - We maintain that existential statements never result in a presupposition failure, following the assertive analysis.
  - Nevertheless, following the presuppositional analysis, we acknowledge that existential statements can sometimes be neither true nor false. We propose to model this (distinct) state of affairs is modeled via a fourth truth-value $\star$.

3 Going quadrivalent

- Launching straight in, the semantics we propose for existential statements is given in (18). We exploit an additional non-truth, non-falsity value $\star$.[10]

(18) **Quadrivalent semantics for existential statements:**

$$[\![\exists_x P(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g), g(x) \in P_w \\ 0 & P_w = \emptyset \\ \star & \text{otherwise} \\ \# & \text{never} \end{cases}$$

- (18) still inherits an issue from the presuppositional analysis: an existential statement is $\star$ at certain possibilities where it's classically true, i.e., those at which x is undefined but $P \neq \emptyset$.
- We therefore an accompanying notion of **truth** relative to (w, g) that eliminates $\star$.[11]
- In order to accomplish this, I'll first introduce a notion of *minimal (non-$\star$) extension.*[12]

[9] Even if this move is considered legitimate, Mandelkern's (2022) account in any case does not offer a straightforward account of cataphoric bathroom sentences.

[10] For existing work in linguistic semantics exploiting a four-valued system, see Muskens 1995.

[11] The definition of **truth** given here differs from the one given in Anvari & Elliott (2025) in non-trivial ways. Anvari & Elliott (2025) define truth at (w, g) by existentially quantifying over *all* assignments that agree with g on the 'domain sensitive' variables, i.e., those variables which, if left undefined, risk #. A definition in terms of minimal extensions seems to harmonize more straightforwardly with a non-eliminative notion of context update.

[12] $g \leq g' \iff \forall x \in dom(g), g(x) = g'(x)$

(19) **Minimal (non-$\star$) extension:**
$Min_w(\phi)(g) = \{\, g' \mid g \leq g', [\![\phi]\!]^{w,g'} \neq \star, \neg\exists h, g \leq h < g', [\![\phi]\!]^h \neq \star \}$

- Some consequences of this definition:

(20) $\exists_x P(x)$

a. $Min_{w_{ab}}(20)([]) = \{\, [x \to a], [x \to b] \,\}$

b. $Min_{w_{ab}}(20)([x \to a]) = \{\, [x \to a] \,\}$ (20) already 1 at $(w_{ab}, [x \to a])$

c. $Min_{w_\emptyset}(20)([]) = \{\, [] \,\}$ (20) already 0 at $(w_\emptyset, [])$

d. $Min_{w_a}, (20)([x \to b]) = \emptyset$ (20) still $\star$ at every extension

- Note that since an atomic sentence such as $P(x)$ is never $\star$, $Min_w(P(x))(g) = \{\, g \,\}$, for any (w, g).[13]

(21) **Truth in a four-valued setting:**
A sentence ϕ is **true** at (w, g) if $\exists h \in Min_w(\phi)(g), [\![\phi]\!]^{w,h} = 1$.

- Appropriate definitions for falsity and presupposition failure will come later.
- An immediate consequence of (21): the truth-conditions of existential statements are a bit more plausible.

(22) $\exists_x P(x)$ is **true** at (w, g) if either:

a. $g(x) \in P_w$

b. $x \notin dom(g)$ and $P_w \neq \emptyset$

- Note that $\exists_x P(x)$ still has a chance to not be true if $P_w \neq \emptyset$, just in case x is mapped to a non-P.
  - My hope is that this case will be independently ruled out by a version of the *novelty* condition on indefinites Heim 1982, discussed further in §6.
- This notion of truth makes good predictions for negative existential statements, assuming that $\star$, like #, is projective.

ϕ	$\neg\phi$
1	0
0	1
#	#
$\star$	$\star$

Figure 3: Quadrivalent negation

(23) $$[\![\neg\exists_x P(x)]\!]^{w,g} = \begin{cases} 1 & P_w = \emptyset \\ 0 & x \in dom(g), g(x) \in P_w \\ \star & \text{otherwise} \end{cases}$$

- (23) is **true** at (w, g) if there are no Ps, irregardless of g.
- It follows that, for any (w, g), s.t., $\neg\exists_x P(x)$ is **true**, $Min_w(\neg\exists_x P(x))(g) = \{\, g \,\}$.
  - This reflects the fact that negation renders an indefinite in its scope inaccessible (Groenendijk & Stokhof 1991).

[13] This is reminiscent of Groenendijk & Stokhof's (1991) notion of a 'test'.

4 Conjunctive sentences

4.1 Accounting for discourse anaphora

- Recall that we assumed a *Middle Kleene* semantics for conjunction.
- Now that we have a fourth truth-value, this needs to be extended.
  - For now, the natural null hypothesis: $\star$ projects just like # (Anvari & Elliott 2025 call this 'Undefinedness Uniformity').

$\phi \wedge^{mk*} \psi$	1	0	#	$\star$
1	1	0	#	$\star$
0	0	0	0	0
#	#	#	#	#
$\star$	$\star$	$\star$	$\star$	$\star$

Figure 4: Middle Kleene conjunction extended

- Let's apply $\wedge^{mk*}$ to discourse anaphora, (24).

(24) There's a^x paradise duck, and it$_x$ quacked. $\exists_x P(x) \wedge^{mk*} Q(x)$

- Some initial observations:
  - If the first conjunct is 1, the second conjunct is never # (since $x \in dom(g)$).
  - If the first conjunct is 0, the whole sentence is 0.
  - If the first conjunct is $\star$, the whole sentence is $\star$.
- Since the first conjunct is never #, the sentence is never #: the presupposition of the pronoun *qua* free variable is filtered.

(25) $$[\![\exists_x P(x) \wedge^{mk*} Q(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g), g(x) \in P_w \cap Q_w \\ 0 & P_w = \emptyset \text{ or } x \in dom(g), g(x) \in P_w - Q_w \\ \star & \text{otherwise} \end{cases}$$

- Given an assignment g, s.t., $x \notin dom(g)$, (24) is thereby **true** at (w, g) if there's a Paradise duck that quacked.
- This case will help us get a handle on the correct notion of **falsity**. Assume a world w_1 where a is a paradise duck that quacked, and b is a paradise duck that didn't quack.
- The sentence $\exists_x P(x) \wedge Q(x)$ is $\star$ at $(w_1, [])$, so we can compute the minimal non-$\star$ extensions of $[]$.

(26) $$Min_{w_1}(\exists_x P(x) \wedge Q(x))([]) = \{ \underbrace{[x \to a]}_{1}, \underbrace{[x \to b]}_{0} \}$$

- Intuitively, the existence of a minimal extension that makes the sentence 0 is not sufficient for **falsity**; the sentence is **false** only if *every* minimal extension makes it 0.

(27) **Truth and falsity in a four-valued setting:**

a. ϕ is **true** at (w, g) if $\exists h \in Min_w(\phi)(g), [\![\phi]\!]^{w,h} = 1$.
b. ϕ is **false** at (w, g) if $\forall h \in Min_w(\phi)(g), [\![\phi]\!]^{w,h} = 0$, and $Min_w(\phi)(g) \neq \emptyset$.
c. ϕ is a presupposition **failure** otherwise.

- Take a world w_2, at which a and b are both paradise ducks that don't quack.

(28) $Min_{w_2}(\exists_x P(x) \wedge Q(x))([]) = \{ \underbrace{[x \to a]}_{0}, \underbrace{[x \to b]}_{0} \}$

4.2 Blocking discourse cataphora

- Its easy to see how cataphora is blocked, given the asymmetric nature of the (extended) Middle Kleene truth-table.
- An immediate consequence of the truth table is that the presupposition of the pronoun qua free variable projects; (29) is # if $x \notin dom(g)$.

(29) It$_x$ quacked and there's a^x Paradise duck. $Q(x) \wedge \exists_x P(x)$

- The semantics for (29) is otherwise not very plausible, since we've yet to incorporate an account of *novelty*; even if its presupposition is satisfied, (29) should nevertheless be unusable (Heim 1982).

(30) $$[\![Q(x) \wedge \exists_x P(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g), g(x) \in Q_w \cap P_w \\ 0 & x \in dom(g), g(x) \in Q_w \to P_w = \emptyset \\ \# & x \notin dom(g) \\ \star & \text{otherwise} \end{cases}$$

- Our idea: The sentence $Q(x) \wedge \exists_x P(x)$ is unusable, because the speaker could've said $Q(x) \wedge P(x)$, which is true in exactly the same cases (and if the presupposition of the former is satisfied, that of the latter must be too).
- To be explored in §6. First, disjunctive sentences.

5 Disjunctive sentences

- The minimally adequate truth table for the purposes of presupposition projection is Strong Kleene, so let's take that as our starting point.
- According to 'Undefinedness Uniformity', we might expect $\star$ to project according to Strong Kleene too.
- This turns out to deliver the wrong results; rather, what we want is for $\star$ to project according to Weak Kleene in disjunctions.

$\phi \vee^{sk*} \psi$	1	0	#	$\star$
1	1	1	1	$\star$
0	1	0	#	$\star$
#	1	#	#	$\star$
$\star$	$\star$	$\star$	$\star$	$\star$

Figure 5: Strong Kleene disjunction extended

- Note that Figure 5 is completely symmetric (in contrast to extended Middle Kleene conjunction); (31) and (32) are expected to be semantically equivalent.
  - As long as we deliver intuitively plausible truth-conditions for the anaphoric case, this will automatically extend to the cataphoric case; a clear advantage of keeping the semantics static.

(31) Either there isn't an^x animal here, or it$_x$ barked. $\neg\exists_x A(x) \vee B(x)$

(32) Either it$_x$ barked, or there isn't an^x animal here. $B(x) \vee \neg\exists_x A(x)$

- It's most straightforward to begin with the falsity clause—since DNE is valid in the four-valued setting, this mirrors the truth clause for $\exists_x A(x) \wedge \neg B(x)$.
  - If the first disjunct is 0, the second is never #, which means that the # clause could only be relevant if the first disjunct is 1, but in this case the entire disjunction is 1.
  - If the first disjunct is 0, the second disjunct being 1 makes the whole disjunction 1.
  - Since $\star$ always projects, the disjunction simply inherits the $\star$ clause from the first disjunct.

(33) $$[\![\neg\exists_x A(x) \vee B(x)]\!]^{w,g} = \begin{cases} 1 & P_w = \emptyset \text{ or } x \in \mathit{dom}(g), g(x) \in A_w \cap B_w \\ 0 & x \in \mathit{dom}(g), g(x) \in A_w - B_w \\ \star & \text{otherwise} \end{cases}$$

5.1 'Weak' readings for disjunction

- This predicts (conditional) existential truth conditions. To see why, consider what is predicted for **truth**. The disjunction can be true in two different ways.
  - It's **true** at (w, g) if there are no animals at w, irregardless of g.
  - Otherwise, it's **true** at (w, g) if there's a minimal extension of g at which it's 1. This will hold just in case there is some animal that barked; the existence of an animal that didn't bark will not falsify the sentence.
- Note that this is different from the universal truth-conditions assumed by Krahmer & Muskens 1995, but it is in-line with the recent accounts of Elliott 2020, Mandelkern 2022 and Spector 2024.
  - See especially Elliott 2024 for a detailed defense of existential truth-conditions.
  - Promissory note: this isn't to say that a universal reading isn't available. more later.

(34) Either Luna doesn't have a^x credit card, or she paid with it$_x$.
true if Luna paid with one of her credit cards

(35) Either Keny doesn't have an^x umbrella, or he remembered to bring it$_x$.
true if Keny remembered to bring one of his umbrellas

5.2 On the status of de Morgan's

- De Morgan's equivalences are not generally valid, but note that the following equivalences do go through.

(36) $[\![\neg\exists_x A(x) \vee B(x)]\!]^{w,g} = [\![\neg(\exists_x A(x) \wedge \neg B(x))]\!]^{w,g}$

(37) $[\![\neg(\neg\exists_x A(x) \vee B(x))]\!]^{w,g} = [\![\exists_x A(x) \wedge \neg B(x)]\!]^{w,g}$ for any world-assignment pair (w, g)

(38) It's not the case that there's both a duck and it didn't quack.
(cf. either there's no duck, or it quacked.)

(39) Neither is there no duck, nor did it quack.
(cf. There's a duck, and it didn't quack)

- Due to variable asymmetries in the semantics of the connectives, the following equivalences don't go through:

(40) $[\![B(x) \vee \neg\exists_x A(x)]\!]^{w,g} \neq [\![\neg(\neg B(x) \wedge \exists_x A(x))]\!]^{w,g}$

(41) $[\![\neg(B(x) \vee \neg\exists_x A(x))]\!]^{w,g} \neq [\![\neg B(x) \wedge \exists_x A(x)]\!]^{w,g}$ for some world-assignment pair (w, g)

(42) ✗ It's not the case that both it$_x$ didn't quack, and there's a^x duck.
(cf. Either it$_x$ quacked, or there's no^x duck)

(43) ✗ It didn't quack, and there's a duck.
(cf. Neither did it quack, nor is there no duck)

5.3 The Strong Kleene alternative

- We'll finish this section by considering an alternative extensions of Strong Kleene disjunction, and consider why it doesn't intuitively deliver the correct result. In Figure 6, $\star$ is assumed to project just like #.

$\phi \veebar^{sk*} \psi$	1	0	#	$\star$
1	1	1	1	1
0	1	0	#	$\star$
#	1	#	#	$\star$
$\star$	1	$\star$	$\star$	$\star$

Figure 6: Strong Kleene disjunction extended (non-canonical)

- To see that this won't work, it's sufficient to merely consider the truth clause for a simple bathroom disjunction $\neg\exists_x A(x) \vee B(x)$.
- This is predicted to be true as soon as x is a B (and x is defined), even if x is not an A! I.e., the anaphoric link between the indefinite and pronoun is not established.

6 Novelty

- An outstanding problem: discourse cataphora is successfully blocked, but the predicted truth-conditions are still implausible; a sentence such as $Q(x) \wedge \exists_x P(x)$ should be *unusable*.

(44) **Heimian (syntactic) novelty:**
no indefinite Noun Phrase is co-indexed with an argument to its left.
after Heim 1982: chapter 2

- Our analysis of cataphoric bathroom disjunctions is incompatible with a syntactic formulation of novelty based on leftness! Only a semanto-pragmatic formulation will do.

(45) Either it$_x$'s upstairs, or there isn't a^x bathroom.

- Another exception to Heimian novelty: Groenendijk & Stokhof's (1991) *program disjunctions* (see also Stone 1992).

(46) A^x professor or an^x assistant professor will attend the meeting of the university board.
He$_x$ will report to the faculty. Groenendijk & Stokhof 1991: p. 88

(47) Either a^x philosopher is in the audience or a^x linguist is.
(Either way) I hope she$_x$ enjoys it. Elliott 2020

- Every analysis of program disjunctions we know of exploits the assumption that they involve *co-indexed* indefinites (see e.g., Groenendijk & Stokhof 1991, Rothschild 2017, Elliott 2020, Heim 2024), and it's difficult to conceive of an alternative.
- An alternative to Heimian novelty: novelty via *Maximize Presupposition!* (as suggested in passing by Heim 2024).
- The idea informally: a sentence with an indefinite of the form 'an^x NP' has an *alternative* derivable via substitution with 'the$_x$ NP'.

(48) An^x P Q-ed $\rightsquigarrow \exists_x[P(x) \wedge Q(x)]$

(49) The$_x$ P Q-ed $\rightsquigarrow \mathit{the}_x(P(x))(Q(x))$

(50) **Semantics of anaphoric definites:**

$$[\![\mathit{the}_x(P(x))(Q(x))]\!]^{w,g} = \begin{cases} 1 & x \in \mathit{dom}(g), x \in P_w \cap Q_w \\ 0 & x \in \mathit{dom}(g), x \in P_w - Q_w \\ \# & \text{otherwise} \end{cases}$$

- N.b., that an anaphoric definite with a tautological restrictor is semantically equivalent to a sentence with a free variable, so for simplicity we can assume that $\exists_x P(x)$ competes with $P(x)$.

(51) It$_x$ is a Paradise duck and something^x quacked.

$$[\![P(x) \wedge \exists_x Q(x)]\!]^{w,g} = \begin{cases} 1 & x \in \mathit{dom}(g), g(x) \in P_w \cap Q_w \\ 0 & x \in \mathit{dom}(g), g(x) \in P_w \to Q_w = \emptyset \\ \# & x \notin \mathit{dom}(g) \\ \star & \text{otherwise} \end{cases}$$

(52) It$_x$ is a Paradise duck and it$_x$ quacked.

$$[\![P(x) \wedge Q(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g), g(x) \in P_w \cap Q_w \\ \# & x \notin dom(g) \\ 0 & \text{otherwise} \end{cases}$$

- The idea: in a context where a sentence with an indefinite S and its substitution alternative S' are equally informative, and the presuppositions of S' are satisfied, S is unusable.
- In a context where x is familiar, the presuppositions of the substitution alternative are satisfied, and both sentences convey that $x \in P_w \cap Q_w$, hence S is unusable.
- Note that cataphoric bathroom sentences are predicted to be usable if $x \notin dom(g)$.

(53) $$[\![P(x) \vee \neg\exists_x Q(x)]\!]^{w,g} = \begin{cases} 1 & Q_w = \emptyset \text{ or } x \in dom(g), g(x) \in P_w \cap B_w \\ 0 & x \in dom(g), g(x) \in Q_w - P_w \\ \star & \text{otherwise} \end{cases}$$

(54) $$[\![P(x) \vee \neg Q(x)]\!]^{w,g} = \begin{cases} 1 & x \in dom(g) \text{ and either } g(x) \in P_w \text{ or } g(x) \notin Q_w \\ 0 & x \in dom(g) \text{ and either } g(x) \notin P_w \text{ or } g(x) \in Q_w \\ \# & \text{otherwise} \end{cases}$$

- The case of program disjunction is more involved—we'll leave this for another occasion.

7 Non-eliminative update

- The static, four-valued semantics outlined here can be combined with a dynamic, non-eliminative notion of context update (Heim 1982, Groenendijk & Stokhof 1991, Groenendijk, Stokhof & Veltman 1996, Dekker 1996).
- Let an 'initial context' be one in which every world is paired with the empty assignment [] (no discourse referents have been introduced).

(55) **Context update in a four-valued setting:**

a. A sentence ϕ, asserted relative to a discourse context c, results in an updated context $c[\phi]$.

b. $c[\phi]$ is defined if $\neg\exists(w,g) \in c$ s.t., ϕ is a presupposition failure at (w,g).[14]

c. If defined, $c[\phi] = \{\, (w,h) \mid \exists g, (w,g) \in c, h \in Min_w(\phi)(g), [\![\phi]\!]^{w,h} = 1 \,\}$

- Heimian *familiarity* immediately follows from the bridge principle in (55b); $c[P(x)]$ will be undefined unless x is defined *throughout* c.
- Assertion of an existential statement may introduce a discourse referent, by extending assignments in the input context.
- Consider the following initial context:

(56) $c_1 := \{\, w_{ab}, w_a, w_b, w_\emptyset \,\} \times []$

$\phi := \exists_x P(x)$

a. $Min_{w_{ab}}(\phi)[] = [x \to a, x \to b]$ $\qquad$ ϕ is 1 at $(w_{ab}, [x \to a])$ and $(w_{ab}, [x \to b])$

[14] von Fintel (2008) calls this clause 'Stalnaker's bridge'—it's independently necessary for a trivalent account of presupposition.

b. $Min_{w_a}(\phi)[] = [x \to a]$ ϕ is 1 at $(w_b, [x \to a])$

c. $Min_{w_b}(\phi)[] = [x \to b]$ ϕ is 1 at $(w_b, [x \to b])$

d. $Min_{w_\emptyset}(\phi)[] = []$ ϕ is 0 at $(w_\emptyset, [])$

(57) $c_1[\phi] = \{\, (w_{ab}, [x \to a]), (w_{ab}, [x \to b]), (w_a, [x \to a]), (w_b, [x \to b]) \,\}$

- As a result, we can account for bona fide discourse anaphora, without text-level conjunction.

(58) There's a^x paradise duck. ⇒ *makes x familiar; x is a paradise duck*

(59) It$_x$ quacked. ⇒ *eliminates possibilities at which x doesn't quack*

- Implicit assumption in the literature: non-eliminative update goes hand-in-hand with a genuinely *dynamic* semantic component (see, e.g., Rothschild & Yalcin 2016 for relevant discussion).
- To our knowledge, ours is the first attempt to combine a genuinely *static* semantics with a non-eliminative notion of context update.
  - Much more to explore here!

8 Deriving variable asymmetries

- The four-valued approach is only appealing as insofar as the projection behavior of # can lean on an explanatory trivalent account of presupposition projection.
- We crucially assumed that # projects according to Middle Kleene for conjunction, and according to Strong Kleene for disjunction, in order to achieve descriptive adequacy.
  - Strong Kleene truth-tables can be derived via independently motivated principles for reasoning in the presence of indeterminacy (Beaver & Krahmer 2001).
  - Middle Kleene truth-tables can be derived via an incrementalization of Strong Kleene reasoning (George 2014).
  - This begs the question of whether the variable asymmetry of conjunction/disjunction can be accounted for in a trivalent setting, in a principled way.
- George's (2008b, 2008a) 'disappointment' theory provides a way of doing just this. We'll only provide an informal sketch here of how disappointment works.[15]
- Disappointment leans on an intuition of incrementality: arguments to connectives are evaluated in a way that mirrors linear order.
  - Evaluation procedure for $p \wedge q$:
  - $[\lambda t \,.\, \lambda u \,.\, t \wedge u](p)$
  - $\Rightarrow [\lambda u \,.\, p \wedge u]$
  - $[\lambda u \,.\, p \wedge u](q)$
  - $\Rightarrow p \wedge q$
- For any # argument, we ask if there is any sequence of subsequent arguments that will allow the sentence to be 1, in a Strong Kleene sense.

[15]The only other projection theory that accomplishes this, to my knowledge, is Kalomoiros's (2023) 'limited symmetry'.

- If there isn't, then # is *disappointing* and we throw # as the value of the sentence, otherwise evaluation continues.
- Disappointment derives Middle Kleene for conjunction, since an initial # argument is disappointing:
  - $[\lambda p \,.\, \lambda q \,.\, p \wedge^{sk} q](\#)$
  - $\Rightarrow \lambda q \,.\, \# \wedge^{sk} q$
  - $\neg\exists t \in \{1, 0, \#\}, \# \wedge^{sk} t = 1$, so return #
  - $[\lambda p \,.\, \lambda q \,.\, p \wedge^{sk} q](0)$
  - $\Rightarrow \lambda q \,.\, 0 \wedge^{sk} q$
  - $[\lambda q \,.\, 0 \wedge^{sk} q](\#)$
  - $\Rightarrow 0 \wedge^{sk} \# = 0$
- Disappointment derives Strong Kleene for disjunction, since an initial # argument is never disappointing.
  - $[\lambda p \,.\, \lambda q \,.\, p \vee^{sk} q](\#)$
  - $\Rightarrow \lambda q \,.\, \# \vee^{sk} q$
  - $\exists t \in \{1, 0, \#\}, \# \vee^{sk} t = 1$ (namely, 1), so continue.
- Disappointment provides a principled way of deriving the parts of the truth tables concerning, $\{1, 0, \#\}$, but how do we integrate $\star$?
- Our conjecture: whereas # is interpreted as indeterminacy (resulting in Strong Kleene reasoning), $\star$ is interpreted as bona fide undefinedness (resulting in Weak Kleene reasoning).
- Nevertheless, sometimes $\star$ is ignored due to **minimal evaluation**.

(60) **Undefinedness of $\star$:**
If, at any point during incremental evaluation, a $\star$ argument is encountered, throw $\star$.

(61) **Minimal evaluation:**
If, at any point during incremental evaluation of a complex sentence, *falsity* is guaranteed (for any sequence of subsequent arguments in $\{1, 0, \#\}$), then return 0 without evaluating further.

- The interplay of *disappointment*, *minimal evaluation*, and *undefinedness* ensures that $\star$ projects according to middle Kleene in conjunctions, but according to weak Kleene in disjunctions.
  - Fundamentally, this is because an initial argument of a disjunction can never guarantee falsity, but the initial value of a conjunction can.
- Illustration for conjunction:
  - $[\lambda p \,.\, \lambda q \,.\, p \wedge^{sk} q](0)$
  - $\Rightarrow \lambda q \,.\, 0 \wedge^{sk} q$
  - $\forall t \in \{1, 0, \#\}, 0 \wedge^{sk} t = 0$, so return 0 regardless of q by *minimal evaluation*
- Illustration for disjunction:
  - $[\lambda p \,.\, \lambda q \,.\, p \vee^{sk} q](0)$
  - $\Rightarrow \lambda q \,.\, 0 \vee^{sk} q$

- $\exists t \in \{1, 0, \#\}, 0 \vee^{sk} t \neq 0$, so continue.
- $[\lambda q \,.\, 0 \vee^{sk} q](\star)$
- $\Rightarrow \star$ by *undefinedness.*

- It's easy to verify that applying this reasoning in all cases results in exactly the quadrivalent truth-tables we assume.

9 Open issues and conclusion

- One of, if not *the* major achievement of dynamic semantics: an account of *donkey anaphora.*

(62) If there's a^x bathroom in this house, then it$_x$'s in a funny place.

(63) Everyone who knows a^x linguist would never ask them$_x$ about their$_x$ work.

- Unfortunately, it's easy to verify that applying George's disappointment algorithm to material implication predicts symmetry, which doesn't seem to be borne out.

$\phi \rightarrow^{sk*} \psi$	1	0	#	$\star$
1	1	0	#	$\star$
0	1	1	1	$\star$
#	1	#	#	$\star$
$\star$	$\star$	$\star$	$\star$	$\star$

Figure 7: Extended Strong Kleene implication

- This is simply because material implication is disjunction-like; the antecedent alone is never sufficient to determine falsity.
- This predicts equivalences such as the following:

(64) a. Either there's no^x bathroom, or it$_x$'s in a funny place.
b. If there's a^x bathroom, then it$_x$'s in a funny place.

(65) a. Either it$_x$ it's upstairs, or there's no bathroom.
b. ✗If it$_x$ isn't upstairs, then there's no^x bathroom.

- Donkey *cataphora* seems to be unattested, so this is clearly not a good prediction; although, this is arguably an artifact of the (independently problematic) material implication semantics for the conditional.
- One major challenge for the future is an explanatory, trivalent theory of projection in conditionals, grounded in a more realistic semantic treatment of conditionals.
- Relatedly, it isn't at all trivial to extend our account to quantificational sentences; it remains to be seen whether a trivalent semantics for quantifiers (see, e.g., Beaver & Krahmer 2001 delivers reasonable results in tandem with our four-valued approach.
- Relatedly, so-called 'strong' readings are clearly available for bathroom sentences (Krahmer & Muskens 1995), but our approach only derives 'weak' readings.

(66) Either Keny doesn't have an^x umbrella, or he forgot to bring it$_x$.
⇒ *true only if Keny forgot to bring every one of the umbrellas he owns*

- See Elliott 2024 for a way of deriving strong readings broadly compatible with our semantics, and also Spector 2024 for relevant discussion.
- Zooming out, there is still significant work to be done in understanding how the four-valued approach we've outlined here relates to alternative approaches to anaphora that have emerged in recent years.
- It's interesting to note that Mandelkern's (2022) *bounded* theory is implicitly four-valued (Matt Mandelkern, p.c.)—sentences are evaluated relative to two dimensions, which have two values each.
- Similarly, Elliott's (2020) trivalent dynamic account can be understood as a four-valued relational semantics: a sentence relative to a pair of assignments is either true, false, a presupposition failure, or undefined.
- These parallels suggest a shared, underlying abstraction.

References

Aloni, Maria. 2023. Neglect-Zero Effects in Dynamic Semantics. In Dun Deng et al. (eds.), *Dynamics in Logic and Language*, 1–24. Cham: Springer International Publishing.

Anvari, Amir & Patrick D. Elliott. 2025. Anaphora in a four-valued setting. Unpublished manuscript. MIT/HHU. `https://ling.auf.net/lingbuzz/009360`.

Beaver, David & Emiel Krahmer. 2001. A Partial Account of Presupposition Projection. *Journal of Logic, Language and Information* 10(2). 147–182. `https://doi.org/10.1023/A:1008371413822` (15 November, 2024).

Charlow, Simon. 2020. Static and dynamic exceptional scope. Unpublished draft. To appear in Journal of Semantics (2020). Rutgers University. `https://ling.auf.net/lingbuzz/004650`.

Chatain, Keny. 2025. Reducing pronoun accessibility to presupposition satisfaction. *Journal of Semantics*. ffaf006. `https://doi.org/10.1093/jos/ffaf006` (12 August, 2025).

Chierchia, Gennaro. 1995. *Dynamics of meaning - anaphora, presupposition, and the theory of grammar*. Chicago: University of Chicago Press. 270 pp.

Dekker, Paul. 1996. The Values of Variables in Dynamic Semantics. *Linguistics and Philosophy* 19(3). 211–257. `https://www.jstor.org/stable/25001626` (28 January, 2022).

Elliott, Patrick D. 2020. Towards a principled logic of anaphora. lingbuzz/005562. MIT. `https://ling.auf.net/lingbuzz/005562`.

Elliott, Patrick D. 2023. Partee conjunctions: projection and possibility. Unpublished draft. HHU. `https://ling.auf.net/lingbuzz/006857`. lingbuzz/006857.

Elliott, Patrick D. 2024. Donkey disjunctions and overlapping updates. *Semantics and Linguistic Theory*. 666–685. `https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/33.033` (15 November, 2024).

Evans, Gareth. 1977. Pronouns, Quantifiers, and Relative Clauses (I). *Canadian Journal of Philosophy* 7(3). 467–536. `https://www.jstor.org/stable/40230703` (10 December, 2020).

George, B. R. 2008a. A new predictive theory of presupposition projection. In *Proceedings of SALT 18*, 358–375. Ithaca, NY: Cornell University.

George, B. R. 2008b. Predicting presupposition projection - Some alternatives in the strong Kleene tradition. unpublished manuscript. UCLA. `http://semanticsarchive.net/Archive/DY0YTgxN/`.

George, B. R. 2014. Some remarks on certain trivalent accounts of presupposition projection. *Journal of Applied Non-Classical Logics* 24(1–2). 86–117. https://doi.org/10.1080/11663081.2014.911521 (21 April, 2020).

Gotham, Matthew. 2019. Double negation, excluded middle and accessibility in dynamic semanttics. In Julian J. Schlöder, Dean McHugh & Floris Roelofsen (eds.), *Proceedings of the 22nd Amsterdam Colloquium*, 142–151.

Groenendijk, Jeroen & Martin Stokhof. 1991. Dynamic predicate logic. *Linguistics and Philosophy* 14(1). 39–100. http://www.jstor.org/stable/25001418.

Groenendijk, Jeroen a. G., Martin J. B. Stokhof & Frank J. M. M. Veltman. 1996. Coreference and modality. In *The handbook of contemporary semantic theory* (Blackwell Handbooks in Linguistics), 176–216. Oxford: Blackwell. https://dare.uva.nl/search?identifier=c655089e-9fae-4842-90da-30e46f37825b (22 July, 2020).

Heim, Irene. 1982. *The semantics of definite and indefinite noun phrases*. University of Massachusetts - Amherst dissertation.

Heim, Irene. 1983. On the projection problem for presuppositions. In *Proceedings of WCCFL 2*, 114–125. Stanford University.

Heim, Irene. 2024. Truth value gaps and the innovations of dynamic semantics. Handout from a talk given at the Rolf Schock Prize Symposium in Logic and Philosophy. Stockholm. https://web.mit.edu/heimarchive/docs/Heim-2024-Truth-value_gaps_and_the_innovations_of_dynamic_se.pdf.

Hofmann, Lisa. 2022. *Anaphora and negation*. University of California Santa Cruz dissertation.

Hofmann, Lisa. 2025. Anaphoric accessibility with flat update. *Semantics and Pragmatics* 18. 3:1–69. https://semprag.org/index.php/sp/article/view/sp.18.3 (24 October, 2025).

Kalomoiros, Alexandros. 2023. *Presupposition and its (A-)symmetries*. University of Pennsylvania dissertation. https://www.kalomoiros.com/_files/ugd/159d3d_1dc374c9ffb54093bc4c68acb152b77 pdf.

Kalomoiros, Alexandros & Florian Schwarz. 2024. Presupposition Projection From and vs. or: Experimental Data and Theoretical Implications. *Journal of Semantics* 41(3–4). 331–372. https://doi.org/10.1093/jos/ffae011 (5 August, 2025).

Karttunen, Lauri. 1973. Presuppositions of Compound Sentences. *Linguistic Inquiry* 4(2). 169–193. https://www.jstor.org/stable/4177763 (21 July, 2020).

Krahmer, Emiel & Reinhard Muskens. 1995. Negation and Disjunction in Discourse Representation Theory. *Journal of Semantics* 12(4). 357–376. https://academic.oup.com/jos/article/12/4/357/1728199 (27 June, 2020).

Mandelkern, Matthew. 2022. Witnesses. *Linguistics and Philosophy.*

Mandelkern, Matthew et al. 2020. Weve discovered that projection across conjunction is asymmetric (and it is!) *Linguistics and Philosophy* 43(5). 473–514. http://link.springer.com/10.1007/s10988-019-09276-5 (1 October, 2020).

Muskens, Reinhard. 1995. *Meaning and partiality* (Studies in Logic, Language, and Information). Stanford, Calif. : [Dordrecht?]: CSLI Publications ; FoLLI, European Association for Logic, Language, and Information. 141 pp.

Peters, Stanley. 1979. A truth-conditional formulation of Karttunen's account of presupposition. *Synthese* 40(2). 301–316. https://doi.org/10.1007/BF00485682 (29 July, 2025).

Rothschild, Daniel. 2017. A trivalent approach to anaphora and presupposition. In Alexandre Cremers, Thom van Gessel & Floris Roelofsen (eds.), *Proceedings of the 21st Amsterdam Colloquium*, 1–13.

Rothschild, Daniel & Seth Yalcin. 2016. Three notions of dynamicness in language. *Linguistics and Philosophy* 39(4). 333–355. https://doi.org/10.1007/s10988-016-9188-1 (2 August, 2020).

Schlenker, Philippe. 2008. Be Articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics* 34(3). `https://www.degruyter.com/view/j/thli.2008.34.issue-3/thli.2008.013/thli.2008.013.xml` (13 September, 2020).

Schlenker, Philippe. 2009. Local contexts. *Semantics and Pragmatics* 2. `http://semprag.org/article/view/sp.2.3`.

Schlenker, Philippe. 2010. Local contexts and local meanings. *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 151(1). 115–142. `http://www.jstor.org/stable/40856594`.

Spector, Benjamin. 2024. Trivalence and transparency: a non-dynamic approach to anaphora. Institut Jean Nicod (CNRS - ENS/PSL - EHESS). `https://ling.auf.net/lingbuzz/008396`. lingbuzz/008396.

Stalnaker, Robert. 1976. Propositions. In A. F. MacKay & D. D. Merrill (eds.), 79–91. New Haven: Yale University Press.

Stone, Matthew D. 1992. 'Or' and Anaphora. *Semantics and Linguistic Theory* 2(0). 367–386. `https://journals.linguisticsociety.org/proceedings/index.php/SALT/article/view/3037` (14 November, 2020).

van den Berg, M. H. 1996. Some aspects of the internal structure of discourse. The dynamics of nominal anaphora. `https://dare.uva.nl/search?arno.record.id=7073` (31 August, 2020).

von Fintel, Kai. 2008. What Is Presupposition Accommodation, Again?*. *Philosophical Perspectives* 22(1). 137–170. `https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1520-8583.2008.00144.x` (18 November, 2020).