Explorations in the negative zone

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Roadmap

- Introducing negative individuals: enriching the domain of individuals with a notion of polarity.
- Applying negative individuals to the semantics of numeral expressions.
  - The problem of 'zero'.
  - The problem of unattested negative entailments.
  - van Benthem's problem.
  - Cumulative readings.
- Generalizing the treatment of numerals: all quantificational determiners can be treated as predicates. Applications to:
  - Conservativity.
  - Exceptional scope.
  - *there*-existentials

Individuals and their negative counterparts

- Consider the following sentence:

(1) Ashton but **not Sam** smokes.

- Bledin (2024): If we take the surface syntax at face value, we need a semantics for 'not' that may apply to an individual, and return an individual (building on Akiba 2009, Fine 2017).

- Bledin's conjecture: just as matter stands in relation to anti-matter, **Sam** stands in relation to **not Sam**.
- 'not Sam' is a bona fide negative individual.

(2) $[\![\text{not Sam}]\!] = \mathsf{Sam}^{-}$

- **Sam**$^{-}$ is **Sam**'s 'negative counterpart' (no relation to Lewisian counterpart theory).
- To distinguish ordinary individuals, I'll write **Sam**$^{+}$.
- Individuals are thereby *polarized*.

- Collective conjunction takes two individuals a, b, and forms a group $a \sqcup b$ (Link 1983) ; this extends to *polarized* individuals too.

(3) $$[\![\text{Ashton and not Sam}]\!] = [\![\text{Ashton}]\!] \sqcup [\![\text{not Sam}]\!] = \mathsf{Ashton}^{+} \sqcup \mathsf{Sam}^{-}$$

- An immediate question: what does it mean for a *negative individual* to be true of a predicate?
- Easy, *not Sam* is true of *smokes* iff *Sam* herself is false of *smokes*.

(4) 'Polarized' function application:

a. $[\![\text{smokes}]\!](\mathsf{Ashton}^{+}) \iff$ Ashton smokes

b. $[\![\text{smokes}]\!](\mathsf{Sam}^{-}) \iff$ Sam doesn't smoke

- *Groups* of polarized individuals compose with predicates via an implicit universal quantifier Δ (analogous to Link's distributivity operator):
- Δ applies to a predicate defined for 'ordinary' positive atoms, and returns a predicate of groups of polarized individuals, which universally quantifies over atomic parts.

(5) [Ashton but not Sam] Δ smokes.

$\iff \Delta([\![\text{smokes}]\!])([\![\text{Ashton but not Sam}]\!])$

$\iff [\lambda X . \forall x \sqsubseteq_{At} X, [\![\text{smokes}]\!] (x)](\text{Ashton}^{+} \sqcup \text{Sam}^{-})$

$\iff$ Ashton smokes and Sam does not

- The domain of *polarized* individuals $D^{\pm}$ is built on a 'base' domain D, and then closed under group formation $\sqcup$.
- An important assumption: incoherent groups are filtered out (Akiba 2009).

(6) $D = \{ a, b, c, \ldots \}$

(7) $$D^{\pm} = \left\{ \begin{array}{c} a^+, a^-, b^+, b^-, c^+, c^-, \ldots \\ \cancel{a^+ \sqcup a^-}, a^+ \sqcup b^+, a^+ \sqcup b^-, a^+ \sqcup c^+, a^+ \sqcup c^- \\ a^- \sqcup b^+, a^- \sqcup b^-, a^- \sqcup c^+, a^- \sqcup c^- \\ \ldots \end{array} \right\}$$

- A group of polarized individuals is incoherent if, for any $x \in D$, it contains atomic parts x^+ and x^-.

- Since incoherent groups are filtered out, an interesting property of $D^{\pm}$ is that it has *multiple maximal elements* (with respect to parthood $\sqsubseteq$).
  - Given a base domain $\{a, b, c\}$ the maxima are given below:

(8)
$$\left\{\begin{array}{c} a^{+} \sqcup b^{+} \sqcup c^{+} \\ a^{+} \sqcup b^{+} \sqcup c^{-}, a^{+} \sqcup b^{-} \sqcup c^{+}, a^{-} \sqcup b^{+} \sqcup c^{+} \\ a^{+} \sqcup b^{-} \sqcup c^{-}, a^{-} \sqcup b^{+} \sqcup c^{-}, a^{-} \sqcup b^{-} \sqcup c^{+} \\ a^{-} \sqcup b^{-} \sqcup c^{-} \end{array}\right\}$$

- In developing a semantics for numerals/other quantifiers, reference to such *maximal* elements will be a central component of the analysis.

- Negative individuals might be considered to be ontologically suspect. What exactly are they?
- A more conservative outlook: *a polarized individual is nothing more, and nothing less than an ordinary individual accompanied by a single bit of information* (Bledin 2024, Simon Charlow p.c.).

(9) $\mathsf{Ashton}^{+} := \langle \mathsf{Ashton}, 1 \rangle$

(10) $\mathsf{Sam}^{-} := \langle \mathsf{Sam}, 0 \rangle$

- Polarized individuals can be conceived of as *individual truth-value pairs*; everything I say today can be reconstructed in these terms.

- Following Bledin (2024), I've suggested an enrichment of the domain of individuals: $D^{\pm}$.
- $D^{\pm}$ contains both ordinary 'positive' individuals, negative individuals, as well as groups formed from these things.
- Bledin focuses on 'not' in conjunctions—see Bledin 2024 for further argumentation, focusing on negation and collective conjunction.
- In the following, I will exploit negative individuals *indirectly*, in order to develop a new decompositional semantics for numeral expressions.

Numeral expressions

- There are two standard approaches to the semantics of numeral expressions in the literature.
  - The first treats numerals as Generalized Quantifiers (GQs) (Barwise & Cooper 1981); they count the intersection of the restrictor and the scope.
  - The second treats numerals as *cardinality predicates*; they count atomic parts of groups (Winter 2002).

(11) Three boys smoke.

a. $\#(B \cap S) \geq 3$ GQ theory

b. $\exists X, \text{boys}(X), \#(At^{+}(X)) = 3, \forall x \sqsubseteq_{At} X, x$ smokes
There's a group of boys with three atomic parts, each of whom smokes predicative theory

- Both GQ-theoretic and the cardinality predicate approaches have distinct advantages and disadvantages.
- Problems for GQ-theory: fails to capture the intuition that numeral expressions introduce *groups*.
    - Can't account for plural predication, "three boys met"…
    - …or cumulative readings (to be discussed later).
- The cardinality predicate approach struggles with:
    - Unattested existential entailments for 'less than *n*'.
    - Upper-bounded numerals like 'at most *n*' ('van Benthem's problem').
- My positive proposal can be seen as a hybrid—negative individuals allow for a version of the cardinality predicate theory which maintains the advantages of the GQ-theoretic approach.

- Given a group of polarized individuals, we can extract (a) the *positive atoms*, (b) the *negative atoms*.

(12) a. $At^{+}(a^{+} \sqcup b^{-} \sqcup c^{+}) = \{ a, c \}$

b. $At^{-}(a^{+} \sqcup b^{-} \sqcup c^{+}) = \{ b \}$

c. $At^{\pm}(X) := A^{+}(X) \cup A^{-}(X)$

- Proposal for numerals: numerals count positive atoms; they have a basic *at least* semantics.

(13) $[\![\text{two}]\!]\,(X) \iff \#(At^{+}(X)) \geq 2$

- NPs (crucially) range over *maximal* groups of polarized individuals.

(14) $[\![\text{boy}]\!] = Max_{\sqsubseteq}\{X \in D^{\pm} \mid \forall x \in At^{\pm}(X), x \text{ is a boy}\}$

- Assuming the boys b_1, b_2, b_3, since incoherent pluralities are filtered, there are multiple maxima:

(15) $$[\![\text{boy}]\!] = \left\{\begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array}\right\}$$

- A useful metaphor: each element says, for each boy, whether or not he is true of some yet-to-be-specified predicate P

- Both NPs and numerals are *predicates of groups of polarized individuals*, and therefore may compose via intersective modification (e.g., via Heim & Kratzer's rule of predicate modification).

(16) $$[\![\text{two}]\!] \cap [\![\text{boy}]\!] = \{ X \in D^{\pm} \mid [\![\text{boy}]\!](X), \#(At^{+}(X)) \geq 2 \}$$
$$= \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array} \right\}$$

- *Two* filters out any group with fewer than two positive atoms.

(17) Two boys sneezed.

- We still need to (somehow) compose a *set of groups* with an ordinary predicate of individuals. We adopt a standard technique from the literature on numerals:
    - the NP 'two boys' combines with an implicit existential quantifier ∃ (Partee 1986, Winter 2002).
    - The resulting quantifier composes distributively, via *Δ*.

(18) [∃ two boys] [*Δ* sneezed].

1 iff at least two boys sneezed

$$\lambda P\,.\,\exists X \in \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, \\ b_1^+ \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^+ \sqcup b_3^+ \end{array} \right\}, P(X)$$

$$\lambda X\,.\,\forall x \sqsubseteq_{At} X, [\![\text{sneezed}]\!]\,(x)$$

$\exists$ $\quad [\![\text{two}]\!] \cap [\![\text{boys}]\!]$

two boys

Δ sneezed

- Predicates of groups of *polarized* individuals are expressive enough to account not just for bare numerals, but also for modified numerals.
    - $[\![\text{exactly two}]\!]\,(X) \iff \#(At^{+}(X)) = 2$
    - $[\![\text{less than two}]\!]\,(X) \iff \#(At^{+}(X)) < 2$

(19) $$[\![\text{exactly two}]\!] \cap [\![\text{boys}]\!] = \left\{ \begin{array}{c} \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^+}, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array} \right\}$$

(20) $$[\![\text{less than two}]\!] \cap [\![\text{boys}]\!] = \left\{ \begin{array}{c} \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^+}, \\ \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^+}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^+}, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array} \right\}$$

- While resembling the cardinality predicate account, the polarized account shares a number of advantages with GQ theory:
    - The numeral *zero* is expressible as a cardinality predicate.
    - Unattested existential entailments ‘less than *n*’ are avoided.
    - Issues for upper-bounded numerals like ‘at most/exactly *n*’ don’t arise.
- These points are covered in more detail in the following section.

‘Zero’ as a cardinality predicate

- Numerals as cardinality predicates classically: no plurality has zero atomic parts.
  - ‘zero’ must be treated as a special case, e.g., as a generalized quantifier.
  - Alternatively a special ‘empty’ plurality may be posited (Bylinina & Nouwen 2018, Buccola & Spector 2016).
- In a polarized setting, this isn’t necessary; ‘zero NP’ picks out the unique, maximal, wholly-negative group.

(21) $[\![\text{zero}]\!](X) \iff \#(At^{+}(X)) = 0$

(22) Zero boys sneezed.

$\iff \exists X \in [\![\text{zero boys}]\!], \Delta([\![\text{sneezed}]\!])(X)$

$\iff \exists X \in \{b_1^- \sqcup b_2^- \sqcup b_3^-\}, \Delta([\![\text{sneezed}]\!])(X)$

$\iff \forall x \sqsubseteq_{At} b_1^- \sqcup b_2^- \sqcup b_3^-, [\![\text{sneezed}]\!](x)$

$\iff$ *none of the boys sneezed.*

- The classical account: since no plurality has zero atomic parts, 'less than two boys sneezed' entails that at least one boy sneezed (Buccola & Spector 2016).
- The polarized account: the presence of the wholly-negative group avoids an unattested existential entailment.

(23) $[\![\text{less than two}]\!] \cap [\![\text{boys}]\!] =$

$$\left\{ \begin{array}{c} \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^+}, \\ \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^+}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^+}, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array} \right\}$$

- On the classical view of numerals as cardinality predicates, existential quantification renders upper-bounds inert.
  - The following statements are equivalent:

(24) $\exists X$, X is a group of boys and $\#X = 3$, and X each sneezed.

(25) $\exists X$, X is a group of boys and $\#X \geq 3$, and X each sneezed.

- This precludes a treatment of upper-bounded numerals such as 'exactly *n*' as simple cardinality predicates.

- van Benthem's problem now doesn't arise, since every group encodes *maximal* information.

(26) $[\![\text{exactly two}]\!]\,(X) \iff \#(At^{+}(X)) = 2$

(27) $[\![\text{exactly two}]\!] \cap [\![\text{boys}]\!] =$

$$\left\{ \begin{array}{c} \cancel{b_1^{+} \sqcup b_2^{+} \sqcup b_3^{+}}, \\ b_1^{+} \sqcup b_2^{+} \sqcup b_3^{-}, b_1^{+} \sqcup b_2^{-} \sqcup b_3^{+}, b_1^{-} \sqcup b_2^{+} \sqcup b_3^{+}, \\ \cancel{b_1^{+} \sqcup b_2^{-} \sqcup b_3^{-}}, \cancel{b_1^{-} \sqcup b_2^{+} \sqcup b_3^{-}}, \cancel{b_1^{-} \sqcup b_2^{-} \sqcup b_3^{+}}, \\ \cancel{b_1^{-} \sqcup b_2^{-} \sqcup b_3^{-}} \end{array} \right\}$$

- The upper-bound is encoded already at the level of the DP!

- NP-denotations provide complete answers to the question "which of the NP individuals did P" (for some yet-to-be-specified P).
- Numerals-as-modifiers *restrict* the set of possible answers, by applying a cardinality constraint to the positive atoms.

(28) $[\![\text{two}]\!]\,(X) \iff \#(At^{+}(X)) \geq 2$

(29) $[\![\text{exactly two}]\!]\,(X) \iff \#(At^{+}(X)) = 2$

(30) $[\![\text{less than two}]\!]\,(X) \iff \#(At^{+}(X)) < 2$

(31) $[\![\text{zero}]\!]\,(X) \iff \#(At^{+}(X)) = 0$

- In the following, I'll apply this approach to one of the hardest nuts to crack in numeral semantics: the interaction of upper-boundedness with cumulative readings.

- Plural expressions give rise to *cumulative readings*.

(32) Enrico and Filipe have read b_1, b_2 and b_3.
⇒ *Enrico and Filipe each read one of* $b_{1\ldots3}$.
⇒ *Each of* $b_{1\ldots3}$ *was read by either Enrico or Filipe.*

- N.b., sentences involving multiple plural arguments also have distributive readings; one way of biasing the cumulative reading is to use the modifier '…between them'.

(33) Enrico, Filipe but not Stan have read b_1, b_2, b_3 but not b_4.
⇒ *Enrico and Filipe each read one of* $b_{1...3}$.
⇒ *Each of* $b_{1...3}$ *have been read by either Enrico or Filipe.*
⇒ *Stan hasn't read any of* $b_{1...4}$
⇒ b_4 *hasn't been read by Enrico, Filipe, or Stan.*

- Intuition: negative individuals interact with cumulativity in a characteristic fashion; they lead to complete non-participation inferences.

- Cumulativity standardly encoded by an operator .** (Beck & Sauerland 2000).
  - Conjecture: cumulativity is *polarity sensitive*.

(34) $$\begin{aligned} {}^{**}R(X,Y) \iff\ & \forall x \in At^{+}(X), \exists y \in At^{+}(Y), R(x,y) \\ & \wedge \forall y \in At^{+}(Y), \exists x \in At^{+}(X), R(x,y) \\ & \wedge \neg\exists x \in At^{-}(X), y \in At^{\pm}(Y), R(x,y) \\ & \wedge \neg\exists y \in At^{-}(Y), x \in At^{\pm}(X), R(x,y) \end{aligned}$$

- The first two lines say that $At^{+}(X)$ and $At^{+}(Y)$ stand cumulatively in the R relation.
- The final two lines encode non-participation of negative individuals.

- The problem of cumulative readings with modified numerals:

(35) Exactly two students read exactly three books.

- Truth-conditions, informally:
    - Some students read some books.
    - Tallying up the book-reading students: there are no more, and no less than two.
    - Tallying up the books-read-by-students: there are no more, and no less than three.
- The problem: the upper-bounds imposed by *exactly n* apply globally; they do not scopally interact (Landman 2000).

- Why is this a problem?
  - In a more standard setting, upper-bounds are imposed via a *maximality* condition.
  - Maximality conditions take scope; their interaction with cumulativity seems to be at odds with compositionality (Krifka 1999).
- See especially (Brasoveanu 2013) on why the standard picture fails to derive attested cumulative readings.
- Existing approaches to this problem exploit powerful mechanisms for side-stepping scopal interactions, such as post-suppositions (Brasoveanu 2013, Charlow 2021, Haslinger & Schmitt 2020).

- The current account immediately predicts this interaction; modified numerals are merely existential quantifiers over groups of polarized individuals.
- Existential quantifiers scopally commute.

(36) Exactly two students read exactly two books.
$\exists X \in [\![\text{ex. 2 students}]\!], \exists Y \in [\![\text{ex. 2 books}]\!], ^{**} [\![\text{read}]\!] (X, Y)$

(37) $$\exists X \in \left\{ \begin{array}{c} s_1^+ \sqcup s_2^+ \sqcup s_3^-, \\ s_1^+ \sqcup s_2^- \sqcup s_3^+ \\ s_1^- \sqcup s_2^+ \sqcup s_3^+ \end{array} \right\}, \exists Y \in \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^-, \\ b_1^+ \sqcup b_2^- \sqcup b_3^+ \\ b_1^- \sqcup b_2^+ \sqcup b_3^+ \end{array} \right\}, ^{**} [\![\text{read}]\!] (X, Y)$$

- The predicted truth conditions are disjunctive; given students s_1, s_2, s_3 and books b_1, b_2, b_3, either:
  - s_1, s_2 read b_1, b_2 between them; s_3 didn't read any book, and b_3 wasn't read by anyone, or…
  - … s_1, s_3 read b_1, b_2 between them; s_2 didn't read any book, and b_3 wasn't read by anyone, or…
  - … s_2, s_3 read b_1, b_2 between them; s_1 didn't read any book, and b_3 wasn't read by anyone, etc.
- In general: some students read some books.
  - Tallying up the book-reading students, there are exactly two.
  - Tallying up the books-read-by-students, there are exactly two.
- Since upper-bounds are *encoded in the groups that DPs range over*, no scopal interactions are expected.

- The following example also has a (doubly) *distributive* reading, compatible with four (or more) books having been read.
    - Informally: *exactly two boys are s.t., they each read exactly two books; the other boys read either more than two books, or less than two.*

(38) Exactly two students read exactly two books.

- This follows from allowing modified numerals to independently take distributive scope, as on any theory.

(39) $\exists X \in [\![\text{ex. 2 students}]\!]\ (\Delta(\lambda x\,.\,\exists Y \in [\![\text{ex. 2 books}]\!]\ (\Delta(\lambda y\,.\,R(x,y))(Y))))(X)$

- A novel prediction: *zero* is an ordinary numeral. It should support cumulative readings.

(40) Context: *a college professor has asked their students to submit questions in advance of their weekly seminar, and each student is allowed to submit more than one question. The professor asks their teaching assistant how many questions were submitted (fearing the worst). Their teaching assistant responds:*
Unfortunately, zero students submitted zero questions this week.

- $^{**}S(s_1^- \sqcup s_2^- \sqcup s_3^- \sqcup \ldots, q_1^- \sqcup q_2^- \sqcup q_3^- \sqcup \ldots)$

- Some naturally occurring examples:

(41) "Dear Reader, you need to unplug yourself because nobody outside Reddit cares or even knows about what arguments are going on there. I've met **zero** people in **zero** places in meat space that have heard of Rippetoe or Mehdi, […] or any other God we cared to hold up…" (my emphasis)
`https://web.archive.org/web/20250212095935/https://purplespengler.blogspot.com/`

(42) "At the time btw **zero** people gave **zero** fucks about gain-of-function research. Trump cleared that in US between 2017 and 2020." (my emphasis)
`https://web.archive.org/web/20250212100005/https://pokerfraudalert.com/forum/showthread.php?19983-So-coronavirus-is-definitely-going-to-kill-a-few-of-us%252Fpage638`

- We've seen that incorporating polarity in the individual domain allows us to 'upgrade' the semantics of numerals as cardinality predicates.
- Groups of polarized individuals effectively encode maximal information at the level of the DP, allowing for straightforward accounts of data that vexes more standard approaches.
- Why stop at numeral expressions? In the following, I explore the idea that *all determiners* can be characterized as predicates.

Determiners as predicates

- Recall, NPs denote *maximal* groups of polarized individuals.
    - Filtering out incoherent groups results in multiple maxima.

(43) $$[\![\text{boy}]\!] = \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array} \right\}$$

- Each group corresponds to a *complete answer* to the question: 'which of b_1, b_2, b_3 did P?' (for some yet-to-be-specified predicate P).

- Perhaps unsurprisingly, this perspective immediately extends to quantificational determiners more generally.

(44) $[\![\text{some}]\!] = \{ X \in D^{\pm} \mid At^{+}(X) \neq \varnothing \}$

(45) $[\![\text{every}]\!] = \{ X \in D^{\pm} \mid At^{-}(X) = \varnothing \}$

- 'some' is true of any group with at least some positive atoms; 'every' is true of any group with no negative atoms.
  - Much like numerals determiners may compose with NPs as intersective modifiers (e.g., via the rule of *Predicate Modification* (Heim & Kratzer 1998)).

(46) $[\![\text{some}]\!] \cap [\![\text{boy}]\!] =$

$$\left\{\begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array}\right\}$$

(47) Some boy sneezed.

$\exists X \in [\![\text{some}]\!] \cap [\![\text{boy}]\!], \Delta([\![\text{sneezed}]\!])(X)$

$\iff \exists x[x \text{ is a boy and } x \text{ sneezed}]$

(48) $[\![\text{every}]\!] \cap [\![\text{boy}]\!] =$

$$\left\{\begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^+}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^+}, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array}\right\}$$

(49) Every boy sneezed.

$\exists X \in [\![\text{every}]\!] \cap [\![\text{boy}]\!], \Delta([\![\text{sneezed}]\!])(X)$

$\iff \Delta([\![\text{sneezed}]\!])(b_1^+ \sqcup b_2^+ \sqcup b_3^+)$

$\iff \forall x[x \text{ is a boy} \rightarrow x \text{ sneezed}]$

- N.b., universal quantification *requires* reference to negative atoms.

- 'no' and 'not every' are straightforwardly definable via predicate negation of 'some' and 'every' respectively.

(50) $[\![\text{no}]\!] = \{ X \in D^{\pm} \mid At^{+}(X) = \varnothing \}$

(51) $[\![\text{not every}]\!] = \{ X \in D^{\pm} \mid At^{-}(X) \neq \varnothing \}$

- I'll leave it as an exercise to verify that these entries result in the correct truth conditions.
  - Intersecting 'no' with an NP denotation results in a singleton set: the maximal, wholly-negative group.
  - Intersecting 'not every' with an NP denotation results in a non-singleton set.

- The standard 'textbook' approach to the semantics of determiners is provided by *Generalized Quantifier theory* (Barwise & Cooper 1981, Westerståhl 2024).
  - In GQ theory, determiner meanings are modeled as relations between sets or (equivalently) as higher-order functions (Montague 1973, Heim & Kratzer 1998).

(52) *Det* $\overbrace{\text{boy}}^{A}$ $\overbrace{\text{sneezed}}^{B}$.

(53) $\textbf{some}(A, B) \iff A \cap B \neq \varnothing$

(54) $\textbf{every}(A, B) \iff A \subseteq B$

(55) $\textbf{no}(A, B) \iff A \cap B = \varnothing$

- Some determiners are famously not first-order definable, and require the expressivity of GQ theory, e.g., proportional 'most'.
- Question: Is the expressivity of the predicative theory limited to first-order definable determiners?

(56) Most boys sneezed.

- In GQ theory:

(57) $\textbf{most}(A, B) \iff \#(A \cap B) > \#(A - B)$
Informally: *the As that B out-number the As that do not B.*

- In the predicative theory, the entry for 'most' is even simpler:

(58) $[\![\text{most}]\!] = \{ X \mid \#(At^{+}(X)) > \#(At^{-}(X)) \}$

- Given an NP *A*, 'most' restricts the range of possible true answers to a question of the form 'which of the *A*s did *B*?'
  - It insists that any true answer entails that the *A*s that did *B* outweigh the *A*s that didn't do *B*.

(59) $[\![\text{most}]\!] \cap [\![\text{boy}]\!] =$

$$\left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array} \right\}$$

- In fact, we can state a mapping from GQ-theoretic determiners to predicates.
    - Let *Det* be a relation between sets.

(60) $Det_{Pred} := \{ X \in D^{\pm} \mid Det(At^{\pm}(X), At^{+}(X)) \}$

- (This mapping is truth-preserving only for *conservative* GQ-theoretic determiners; more on this in a bit).

Illustration

- $\textbf{most}_{Pred} := \{ X \in D^{\pm} \mid \textbf{Most}(At^{\pm}(X), At^{+}(X)) \}$
  - $= \{ X \in D^{\pm} \mid \#(At^{\pm}(X) \cap At^{+}(X)) > \#(At^{\pm}(X) - At^{+}(X) \}$
  - $= \{ X \in D\pm \mid \#(At^{+}(X)) > \#(At^{-}(X)) \}$
- $\textbf{some}_{Pred} := \{ X \in D^{\pm} \mid \textbf{some}(At^{\pm}(X), At^{+}(X)) \}$
  - $= \{ X \in D^{\pm} \mid A^{\pm}(X) \cap A^{+}(X) \neq \varnothing \}$
  - $= \{ X \in D\pm \mid At^{+}(X) \neq \varnothing \}$
- $\textbf{every}_{Pred} := \{ X \in D^{\pm} \mid \textbf{every}(At^{\pm}(X), At^{+}(X)) \}$
  - $= \{ X \in D^{\pm} \mid A^{\pm}(X) \subseteq A^{+}(X) \}$
  - $= \{ X \in D\pm \mid At^{-}(X) = \varnothing \}$

Applications

- In the following, applications of the predicative theory to:
    - The *conservativity* universal.
    - (Exceptional) quantificational scope.
    - Semantically singular vs. semantically plural quantifiers.
    - Restrictions on *there*-existentials.

- A natural question that arises at this point: how expressive is the predicative theory, compared to GQ theory?
- A famous linguistic universal: determiners in natural language express conservative GQ-theoretic determiners (Keenan & Stavi 1986).
- A GQ-theoretic determiner *Det* is conservative iff the following equivalence holds:

(61) $Det(A,B) \iff Det(A, A \cap B)$

- Informally, the non-*A* *B*s are never relevant for determining the truth of a quantificational statement '$Det(A,B)$', if *Det* is conservative.

- In GQ theory, it's straightforward to define non-conservative determiners.
  - A well-known example: the cardinality determiner I.

(62) $I(A, B) \iff \#A = \#B$

- Intuitively, this violates conservativity, since the entirety of B is relevant to determining the truth of the statement, not just $A \cap B$.
- It's easiest to appreciate this with a simple illustration:
  - Let A and B be disjoint singleton sets $\{ a \}$ and $\{ b \}$, so $A \cap B = \varnothing$, $\#A = \#B = 1$.
  - $I(A, B)$ is true in this scenario, but $I(A, A \cap B)$ is false.

- The very fact that non-conservative determiners are stateable in GQ-theory leads to the question: why are they never lexicalized?
    - A tentative suggestion: non-conservative determiners are never lexicalized, because determiners aren't GQ-theoretic relations, but rather predicates of groups of polarized individuals.
    - Non-conservative determiners simply aren't stateable in the predicative theory.
- One way to appreciate this: apply the recipe for mapping GQ-theoretic dets to preds to *I*.

- Assume: $A = \{ a \}; B = \{ b \}$, so $I(A, B)$ is true.

(63) $I_{Pred} := \{ X \in D^{\pm} \mid I(At^{\pm}(X), At^{+}(X)) \}$

(64) $I_{Pred} := \{ X \in D^{\pm} \mid \#At^{\pm}(X) = \#At^{+}(X) \}$

(65) $I_{Pred} \cap A = \{ a^{+} \}$

(66) $B(a^{+}) = 0$

- Informally, I_{Pred} demands that the cardinality of A is the same as the cardinality of $A \cap B$!
  - The mapping is not truth-preserving.
  - Fact: the mapping is *only* truth-preserving for conservative GQ-theoretic determiners.

- The predicative theory has the resources to express *all (and only) the conservative GQ-theoretic determiners*.
- Since determiners-as-predicates compose with A (the NP restrictor) as intersective modifiers, they have the capacity to:
    - place restrictions on A, i.e., $At^{\pm}(X)$.
    - place restrictions on $A \cap B$, i.e., $At^{+}(X)$.
    - place restrictions on $A - B$, i.e., $At^{-}(X)$.
- This is because elements in the denotation of the NP restrictor correspond to complete answers to the question "which of the As did B?".
- Determiners may only restrict this set; B-elements that are not also As simply cannot be affected.

- To complete the picture, we can also map back from predicates to (conservative) GQ-theoretic determiners.
  - Given a predicate Det_{Pred} the corresponding GQ-theoretic determiner is defined as follows.

(67) $Det_{GQ}(A, B)$

$$:= \exists X \in Det_{Pred} \cap Max_{\sqsubseteq}\{X \in D^{\pm} \mid \forall x \in At^{\pm}(X), x \in A\}, \Delta(B)(X)$$

- The headline here: the predicative theory allows us to characterize a *subset* of the determiner meanings expressible in GQ-theory.
- Namely, only those corresponding to the *conservative* GQ-theoretic determiners.

- The resulting picture for the syntax-semantics interface ends up looking a little bit different.
  - DPs have two scope-taking components: *existential scope* and *distributive scope*.
  - Essentially this generalizes machinery from the literature on numeral semantics to all determiners.

(68) Yasu likes some student.

$\exists$ [some student] $\lambda X\ t_X\ \Delta$ [λx Yasu likes t_x]

$\Rightarrow \exists X \in [\![\text{some student}]\!], \Delta(\lambda x\,.\,\text{Yasu likes } x)(X)$

- Scopal ambiguities arise because, even though existential quantifiers commute, the distributive quantifier Δ does not.
    - To illustrate, I'll go through how to derive an inverse scope reading.

(69) Some boy danced with every girl.

$\exists$ [every girl] $\lambda G\ t_G\ \Delta$ [λg . $\exists$ [some boy] $\lambda B\ t_B\ \Delta$ [$\lambda b\ t_b$ danced with t_g]]

$\Rightarrow \exists G \in [\![\text{every girl}]\!], \Delta(\lambda g . \exists B \in [\![\text{some boy}]\!], \Delta(\lambda b . D(b, g))(B))(G)$

$\Rightarrow \exists G \in [\![\text{every girl}]\!], \Delta(\lambda g . \exists b, b$ is a boy and b danced with $g)(G)$

$\Rightarrow \forall g, g$ is a girl, $\exists b, b$ is a boy and b danced with g

- As discussed in the literature on numeral semantics, the existential and distributive scope of a numeral may be *split*.
    - The existential component takes exceptional scope, whereas the distributive component is strictly clause-bounded (Ruys 1992).

(70) If three relatives of mine die, I'll inherit a fortune.

- if..then $>$ $\exists$ $>$ Δ.
    - *If any three relatives of mine all die, I'll inherit a fortune.*
- $\exists$ $>$ if..then $>$ Δ
    - *if a particular three relatives of mine all die, I'll inherit a fortune.*

- If we generalize this machinery to *all* determiners, don't we (incorrectly) predict that, e.g., 'every', and 'no' give rise to exceptional scope readings?

(71) If every relative of mine dies, I'll inherit a fortune.

a. ✓if..then > $\forall$

b. ✗$\forall$ > if..then

(72) If no relative of mine dies, I'll be penniless.

a. ✓if..then > $\neg\exists$

b. ✗$\neg\exists$ > if..then

- Surprisingly, no! Since 'every NP' and 'no NP' denote *singleton sets* on the predicative theory, exceptional existential scope is *vacuous* (assuming the NP restrictor is interpreted transparently!).
- Assume relatives of mine r_1, r_2, r_3.

(73) If every relative of mine dies, I'll inherit a fortune.
$\exists X \in \{ r_1^+ \sqcup r_2^+ \sqcup r_3^+ \}$ if $\Delta(\llbracket \text{dies} \rrbracket)(X)$, I'll inherit a fortune.
$\iff$ if $\Delta(\llbracket \text{dies} \rrbracket)(r_1^+ \sqcup r_2^+ \sqcup r_3^+)$, I'll inherit a fortune.

(74) If no relative of mine dies, I'll be penniless.
$\exists X \in \{ r_1^- \sqcup r_2^- \sqcup r_3^- \}$ if $\Delta(\llbracket \text{dies} \rrbracket)(X)$, I'll be penniless.
$\iff$ if $\Delta(\llbracket \text{dies} \rrbracket)(r_1^- \sqcup r_2^- \sqcup r_3^-)$, I'll be penniless.

- See Schwarzschild (2002) on the vacuity of existential quantification over a singleton set.

- Any determiner that results in a non-singleton set is expected to result in detectable exceptional scope readings. This has already been observed for indefinites and bare numerals (Ruys 1992).
- I contend that modified numerals allow for exceptional scope readings too (contra Cresti 1995, Reinhart 1997, Ruys & Spector 2017).

(75) Context: *James is playing a variant of roulette involving two balls. The rules are simple: James can bet on any two numbers, and if the balls both land on the numbers James picked, he doubles his bet. In any other scenario, James loses his bet. Unbeknownst to James, the croupier has rigged the wheel: the two balls are guaranteed to land on predetermined numbers.*
If James bets on exactly two of these numbers, He'll double his bet.
(I just don't know which two)

- A prediction that I'm much less sure about: since proportional determiners result in non-singleton sets, they should give rise to exceptional scope readings too.

(76) Context: *James is playing a variant of roulette involving a single ball. He can bet on any amount of numbers, but the more numbers he bets on, the smaller his potential winnings. Unbeknownst to James, the croupier has rigged the wheel: the wheel includes numbers 1-20, and the ball is guaranteed to land on 1-15.*
?If James bets on most of these numbers, he'll at least win something.
(I just don't know which ones exactly)

Another loose end: semantic singularity/plurality

- The predicative theory as it stands has nothing to say about, e.g., *some boys* vs. *some boy*.
- Relatedly, it's unclear how to generalize this account to *collective predication*.

(77) *Some boy gathered.

(78) Some boys gathered.

- Note that the predicative theory is no worse off than GQ-theory in this regard.

- One possibility: following (Bledin 2024), assume that the *base domain* is inherently plural.

(79) $^{*}D = \{\, a, b, c, a \oplus b, a \oplus c, b \oplus c, a \oplus b \oplus c, \ldots \,\}$

- Polarizing a plural base domain results in a rich, multi-layered structure:

(80)
$$^{*}D^{\pm} = \left\{ \begin{array}{c} a^{+}, a^{-}, b^{+}, b^{-}, c^{+}, c^{-} \\ (a \oplus b)^{+}, (a \oplus b)^{-}, \ldots \\ a^{+} \sqcup (a \oplus b)^{+}, a^{+} \sqcup (a \oplus b)^{-}, \ldots \\ a^{-} \sqcup (a \oplus b)^{+}, a^{-} \sqcup (a \oplus b)^{-} \ldots \end{array} \right\}$$

- Note that, e.g., $a^{-} \sqcup (a \oplus b)^{+}$ does not count as incoherent.

- Plural NPs denote *maximal* groups of polarized *pluralities*.
    - Let the boys be a, b, c (I omit singletons here for exposition).

(81) $$[\![\text{boys}]\!] = \left\{ \begin{array}{c} (a \oplus b)^+ \sqcup (a \oplus c)^- \sqcup (b \oplus c)^- \sqcup (a \oplus b \oplus c)^- \\ (a \oplus b)^- \sqcup (a \oplus c)^+ \sqcup (b \oplus c)^- \sqcup (a \oplus b \oplus c)^- \\ (a \oplus b)^- \sqcup (a \oplus c)^- \sqcup (b \oplus c)^+ \sqcup (a \oplus b \oplus c)^- \\ \ldots \end{array} \right\}$$

- Collective predicates are true of non-atomic pluralities, such as $a \oplus b$.

(82) Some boys gathered.
$\exists X \in [\![\text{boys}]\!], At^{+}(X) \neq \varnothing, \Delta([\![\text{gathered}]\!])(X)$
$\iff$ *there's a plurality of boys X, s.t., X gathered.*

- Singular 'some boy' is incompatible with a collective predicate such as 'gathered', because it denotes a set of groups of polarized *atoms*.
    - To address a possible point of confusion: since the system is multi-layered, $At^{+}(X)$ may return a set of non-atomic individuals.

- The puzzle: *there*-existentials compatible with only some quantificational associates (Barwise & Cooper 1981).

(83) There are some dogs on the train.

(84) There are no dogs on the train.

(85) #There are all dogs on the train.

- **Good:** bare numerals ('three') modified numerals ('at least/at most/less than/exactly three'), indefinites, negative indefinites, 'many', 'few', …
- **Bad:** universals ('every', 'all', …), proportionals ('most', 'half',…), 'neither', 'both', 'the', …

- **Question:** is the class of determiners that are allowed semantically specifiable in a polarized setting, in a natural and elegant way?
- This has already been done (famously) from a GQ-theoretic perspective (Barwise & Cooper 1981).
- In a polarized settings, the relevant class of determiners can be classified according to their invariance properties.

- A *Det* is negative-invariant if it doesn't care about the presence of negative elements in its argument.

Negative invariance

A determiner *Det* is negative invariant if:

$\forall X, X' \in D^{\pm}, X^{+} = X'^{+}, Det(X) \iff Det(X')$

- It's easy to see that determiners only placing conditions on X^{+} are negative invariant:
  - $[\![\text{some}]\!](X) \iff X^{+} \neq \varnothing$
  - $[\![\text{no}]\!](X) \iff X^{+} = \varnothing$
  - $[\![\text{three}]\!](X) \iff X^{+} \geq 3$
  - $[\![\text{ex. three}]\!](X) \iff X^{+} = 3$

- Varying negative individuals in the make-up of X never affects the truth of a negative-invariant determiner:

(86) $[\![$ex. three$]\!]$ is **true** of $a^+ \sqcup b^+ \sqcup c^+$

$a^+ \sqcup b^+ \sqcup c^+ \sqcup d^-$

$a^+ \sqcup b^+ \sqcup c^+ \sqcup d^- \sqcup e^-$ ...

(87) $[\![$ex. three$]\!]$ is **false** of $a^+ \sqcup b^+$

$a^+ \sqcup b^+ \sqcup c^-$

$a^+ \sqcup b^+ \sqcup c^- \sqcup d^-$ ...

- A *Det* is positive-invariant if it doesn't care about the presence of positive elements in its arguments.

Positive invariance

A determiner *Det* is positive invariant if:

$\forall X, X' \in D^{\pm}, X^- = X'^-, Det(X) \iff Det(X')$

- As far as I can tell, universals are the only positive invariant determiners *lexicalized* in natural language:
  - $[\![\text{every}]\!]\,(X) \iff X^- = \varnothing$

- It's easy to come up with other positive invariant determiners, which, while conservative, aren't intuitively plausible (primitive) determiner meanings.
  - $[\![\text{not every}]\!](X) \iff X^{-} \neq \varnothing$
  - $\textbf{three-not}(X) \iff X^{-} \geq 3$

(88) $\exists X, \textbf{three-not}(X), \Delta([\![\text{sneeze}]\!])(X) \iff$
at least three people didn't sneeze

- Note that the positive-invariant determiners are also *non-negative-invariant*.

- Alongside negative- and positive-invariant determiners, we also encounter determiners that are *neither negative- nor positive invariant*.
    - This includes, most prominently, proportional determiners.

(89) $[\![\text{most}]\!]\,(X) \iff \#X^{+} > \#X^{-}$

(90) $[\![\text{half}]\!]\,(X) \iff \#X^{+} = \#X^{-}$

	negative-invariant	positive-invariant
some	✓	✗
three	✓	✗
exactly three	✓	✗
no	✓	✗
every	✗	✓
most	✗	✗
half	✗	✗
$D_{\top}$	✓	✓
$D_{\bot}$	✓	✓

Note: the only determiners that are *both* negative- and positive invariant are trivial.

- Conjecture: only *negative-invariant* determiners are possible in a *there*-existential.

(91) There are some/three/exactly three/no boys outside.

(92) *There are all/most/half boys outside.

- A natural question arises: Is there a natural counterpart to positive/negative invariance in GQ-theory.
  - Yes. Negative invariance corresponds directly to left conservativity.

- In GQ-theory, negative invariance (I think!) corresponds to left conservativity.

(93) **Left conservativity:**
$D(A)(B) \iff D(A \cap B)(B)$

- Intuitively, this is because positive individuals stand as proxies for restrictor individuals true of the scope.
- It's easy to see that left conservativity fails for a determiner like *every*, since the existence of elements in $A - B$ can falsify the universal statement.

- The positive case is more interesting. It corresponds to the following property in GQ-theory:

(94) Left overlap blindness:
$D(A)(B) \iff D(A - B)(B)$

- The only natural language determiners that satisfy this property (counter-intuitively), are universals.

(95) Every apple is green.

(96) Every non-green apple is green.

- These sentences are logically equivalent…

- Let's work through this in detail.
- First, let's assume that apples $\subseteq$ green, so the first 'every' statement is true.
  - It follows that apples $-$ green $= \varnothing$
  - Therefore 'every non-green apple is green' is (trivially) true, since the restrictor is empty. Moreover, this is the *only* way this statement can be true.
- Now, let's assume that apples $\nsubseteq$ green, so the first 'every' statement is false.
  - It follows that apples $-$ green $\neq \varnothing$.
- Therefore, 'every non-green apple is green' is (trivially) false.
- More generally, it's easy to show that $A \subseteq B \iff (A - B) \subseteq B$.

- Negative-invariance provides a straightforward way of isolating the determiners which are compatible with *there*-existentials.
- However, it still leaves the following deeper question unanswered:
    - *Why* do *there*-existentials only tolerate negative-invariant determiners?
- Furthermore, why is positive-invariance such a marked property?

- Universals seem to be the only positive-invariant determiners that are lexicalized in natural language.
- There are many conservative, positive-invariant determiners that aren't lexicalized. Why?
- A conjecture: the lexicalization of determiner meanings betrays a *positive bias* in natural language; determiner meanings *can* care about the presence/absence of negative individuals, but may not place constraints on them alone over and above this.

Conclusion

- I've proposed that enriching the domain of individuals with a notion of *polarity* solves a range of problems in the semantics of numeral expressions.
- This picture can be more radically extended to quantificational determiners more generally.
- Treating determiners as *predicates* of groups of polarized individuals constitutes a bona fide alternative to Generalized Quantifier theory, the prevailing approach to the semantics of DPs since the 70s.
- Many consequences and applications have yet to be explored - this is only the beginning!

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Any mistakes are my own.

Fin

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