Partee conjunctions

and free choice with anaphora

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1 Introduction

In this talk, I'll be exploring the interaction between epistemic modality and anaphora, as well as the (epistemic) modal-disjunction connection.[1]

I'll address two interconnected topics. The first involves two (non-obvious) claims about the dynamics of possibility statements.

- **Externally dynamic $\Diamond$:** A possibility statement $\Diamond\phi$ introduces anaphoric information in (roughly) the same way as the disjunctive statement $\phi \vee \neg\phi$.
- **Weak satisfaction conditions for possibility statements:** Epistemic modals are (again, roughly) Strong Kleene existential quantifiers over the local context, often giving rise to relatively weak satisfaction conditions.

Let's anticipate some immediate objections - both claims fly in the face of the simplest data involving possibility statements and anaphora.

With regard to the first claim, possibility statements don't seem to introduce anaphoric information at all(1).

(1) John might have a^x car. #It$_x$'s red.

We'll argue that this is just because the open sentence *it$_x$'s red* presupposes that x is *familiar* - whereas the discourse referent introduced by the possibility statement is (initially) merely possible.[2]

[1] An important precursor to this work: (Groenendijk, Stokhof & Veltman 1996).

[2] Ultimately, the argument will be highly reminiscent of the Rothschild's point that the impossibility of discourse anaphora is consistent with disjunctive sentences being externally static (Rothschild 2017, Elliott 2020).

And possibility modals are *holes*[3] ... in fact possibility statements are often used as diagnostics for presuppositionality for this very reason. Upon accepting (2) we typically accommodate the information that there is a (unique) bathroom if it's not already contextually entailed.

(2) The bathroom might be upstairs.
$\overset{?}{\Rightarrow}$ *there is a bathroom*

This will be a little trickier to address, but we'll be inexorably drawn to weak satisfaction conditions by (i) classical filtration diagnostics, and (ii) a new empirical observation I dub *Partee conjunctions* (introduced in the next section).

The dynamics of possibility statements I argue for will, perhaps surprisingly, have important ramifications for the theory of *Free Choice* (Kamp 1973), insofar as we'll be able to address a problem which afflicts any account of free choice stated in terms of disjunctive simplication (which is *almost* all of them).

In order to get the story for epistemic modals off the ground, and predict the right interactions with anaphora, we'll ultimately develop a *bilateral update semantics*, keeping track of both the positive and negative updating information states.[4]

Before we get to the analysis, we'll start with one of the key empirical observations: *Partee disjunctions.*

2 Partee conjunctions

First, some relatively uncontroversial facts about presupposition projection.

- The presupposition of a latter conjunct fails to project if contextually entailed by the initial conjunct.
- Epistemic modals are holes for presupposition projection (as discussed in the introduction).

Partee conjunctions[5] are seemingly incompatible with both:

(3) It's maybe there's no bathroom, and maybe it's upstairs!

[3] In the sense of (Karttunen 1973).

[4] The semantics we'll end up with will be a systematic lifting of Elliott's Strong Kleene dynamic semantics into an update setting; see also (Willer 2018, 2019) for an application of a bilateral update semantics to epistemic modals.

[5] I dub sentences of the form $\Diamond\neg\exists_x P(x) \wedge \Diamond Q(x)$ *Partee conjunctions* since they parallel and are directly inspired by Partee's famous examples involving disjunction and presupposition projection/anaphora ("Either there's no bathroom, or it's upstairs."). Partee *disjunctions* will have an important role to play in the following discussion.

(4) It's possible there's no homework, and it's (equally) possible that it hasn't been assigned yet.

The surprising fact is that bona fide discourse anaphora is possible. That this is really discourse anaphora can be appreciated by considering variations on Heim's sage plant sentences.[6]

(5) It's possible that Sally didn't buy a^x sage plant, and it's possible she bought 8 others along with it$_x$.

(6) Jameson might not have bought a$_x$ drink, and she might've bought another one right after it$_x$.

Partee conjunctions are sentences of the form $\Diamond\neg\exists_x P(x) \wedge \Diamond Q(x)$, in which surprisingly x in the second conjunct is bound.

Although I won't go into details here, the same fact holds for (non-anaphoric) presuppositions.[7] Note that here, local accommodation could be playing a role, but to my ear (8) requires one to accommodate that Al at some point smoked.

(7) It's possible Al never smoked, and it's possible he recently stopped.

(8) It's possible Al is unwell, and it's possible he recently stopped smoking.

These facts are doubly surprising from the perspective of every theory of anaphora/presupposition-projection i'm aware of, dynamic or otherwise.

- An open sentence "it$_x$'s upstairs" should require x to be locally familiar; the local context in "maybe there's no bathroom and maybe it's upstairs", is the first conjunct, which doesn't entail the existence of any bathroom.
- In the majority of dynamic approaches to anaphora, negation typically closes off discourse referents introduced in its scope.[8]

There are two things that are pertinent to mention at this point.

Firstly, the projection pattern we observe in Partee conjunctions patterns with the projection patterns we observe in disjunctions (as Partee famously observed). This modal-disjunction correspondence will be the key to unlocking *anaphora with free choice* (discussed in the following section).

[6] I take failure of uniqueness to be definitional of discourse anaphora; (Heim 1982).

[7] And our analysis, suitably extended, will account for these cases too. The utility of focusing on the anaphoric cases is that we can independently rule out local accommodation.

[8] See, e.g., (Groenendijk & Stokhof 1991). For some notable exceptions see (Krahmer & Muskens 1995, Gotham 2019, Mandelkern 2022, Hofmann 2019, 2022, Elliott 2020).

(9) Either there is no^x bathroom or it$_x$'s upstairs.

Secondly, you may be suspicious that Partee conjunctions are exceptional due to the (in my view, quite poorly understood) mechanism of modal subordination (Roberts 1989), where intuitively a modal can be anaphoric on a body of information introduced by a previous modal.

(10) There might$^\alpha$ be a^x bathroom. It$_x$ would$_\alpha$ be upstairs.

This seems unlikely, since it would require the first modal to introduce the negation of its prejacent as a discourse referent (in some sense). This doesn't seem to be generally possible:

(11) There might$^\alpha$ be a bathroom. I would$_\alpha$ be disappointed.
$\not\Rightarrow$ *If there is no bathroom, I'm disappointed*

I'll suggest that the key to accounting for Partee conjunctions is to recognize that they behave *variably* like disjunction/conjunction with respect to anaphora and projection.

(12) Maybe there's a^x bathroom, and maybe it$_x$'s upstairs.

(13) It's possible Al smoked, and it's possible he recently stopped.

(14) Jameson might have bought a^x drink, and she might've bought another one right after it$_x$.

Modal subordination is harder to rule out as playing a role here, but if we put this to one side for a moment, one straightforward way of interpreting the examples above is as telling us that modalized open sentences have weaker satisfaction conditions than we typically assume.

In fact, other filtration environments bear out this suspicion. The sentences in (15-16) are consistent with $\Diamond\phi_\pi$ presupposing $\Diamond\pi$.[9]

(15) If it's possible there's a^x bathroom, then it's possible the$_x$ bathroom is upstairs.

(16) Either it's certain there's no bathroom, or it's possible the bathroom is upstairs.

It's hard to definitively rule out modal subordination in these cases, but we seem to be missing an obvious generalization. Another case suggesting that modal subordination isn't the right explanation here (at least, without some additional gymnastics):

[9] The logic of (16) may not be immediately clear, but the idea is as follows: the local context of the second disjunct for the purposes of satisfaction is the negation of the first. Here, the first disjunct has the abstract form $\Box\neg\phi$, by duals $\neg\Box\neg\phi \iff \Diamond\phi$.

(17) If [either a^x linguist or a^y philosopher are in the audience], it's possible that the_x linguist hated the talk (and it's possible the_y philosopher loved it).

So much for Partee conjunctions - in the analysis section we'll construct an account which pursues the intuition that epistemic modals filter presuppositions. In the next section I'll briefly introduce a closely related puzzle: *free choice with anaphora.*

3 Free choice with anaphora

Almost all theories of free choice I'm aware, be they semantic or pragmatic, are stated in terms of *disjunctive simplification.*[10]

In other words, to capture the inference from $\Diamond(\phi \vee \psi) \Rightarrow \Diamond\phi \wedge \Diamond\psi$ the vast majority of accounts assume that we reason about alternatives derived from the individual disjuncts (Fox 2007, Bar-Lev 2018, Bar-Lev & Fox 2017), or otherwise place constraints on the individual disjuncts (Zimmermann 2000, Willer 2018, 2019, Goldstein 2019).

A very general for theories of free choice based on *simplification* is posed by *free choice with anaphora.*

(18) You're allowed to write no squib, or submit it on the last day of class.
  a. *You're allowed to write no squib.*
  b. *You're allowed to write a squib and submit it on the last day of class.*

(19) Jameson might not have bought a drink, or bought another one right after it.
  a. *Jameson might not have bought a drink.*
  b. *Jameson might have bought a drink and bought another one right after it.*

In each case, the second disjunct is an open sentence - in order to derive the attested FC inferences, we want to consider the second disjunct *in its local context.*

Intuitively, we want to derive the following result. If we have a logic of anaphora that validates double-negation elimination the rest should be taken care of.

Fact 3.1. ***FC with anaphora:*** $\Diamond(\phi \vee \psi) \Rightarrow \Diamond\phi \wedge \Diamond(\neg\phi \wedge \psi)$

The connection with Partee conjunctions is clear - FC with anaphora entails a corresponding Partee conjunction.

[10]One notable exception is (Aloni 2022).

(20) It's possible that either there's no bathroom or it's upstairs.

(21) $\Rightarrow$ It's possible there's no bathroom, and it's possible it's upstairs.

Eventually, we'll show how once we have a good understanding of Partee conjunctions, the analysis of FC with anaphora will fall neatly into place, but we'll take the long way round.

First, we'll develop an analysis - a *bilateral update semantics* - tailored to account for Partee conjunctions in a principled fashion.

4 Analysis

4.1 Bilateral update semantics

Since we're dealing with *discourse anaphora*, we'll be developing a particular kind of dynamic semantics[11] - namely an *update semantics*, where the meaning of a sentence is stated as the effect it has on a body of information (Heim 1982) (with a bilateral twist).

Treating meanings as updates over information states will be *essential* for incorporating a dynamic treatment of epistemic modals, so this isn't an arbitrary choice (Veltman 1996, Groenendijk, Stokhof & Veltman 1996).

We'll take for granted the datastructures introduced by Heim for encoding both worldly and anaphoric information.

4.1.1 Possibilities and states

Definition 4.1. Possibilities. A possibility is a world-assignment pair (w, g). Assignments are total functions from variables to $D \cup \{\star\}$ (emulating partiality).

Assignment/possibility extension:

- $g \leq h$ iff h agrees with g at every variable that g is 'defined' for.[12]
- $(w, g) \leq (w', h)$ iff $w = w'$ and $g \leq h$

Definition 4.2. Information states. A (Heimian) information state is a set of possibilities. The notion of possibility extension induces a natural ordering between states.

[11] When it comes to discourse anaphora, dynamic semantics is arguably the only game in town. See (Mandelkern & Rothschild 2020) for the most forceful version of the argument that I'm aware of.

[12] Formally: $h \geq g \iff \forall x[g(x) \neq \star \rightarrow h(x) = g(x)]$.

- $s \leq s'$ iff every possibility in s' is an extension of some possibility in s.[13]

This means that an extension of s either (i) eliminates possibilities from s, or (ii) adds anaphoric information to s.

- The unique *ignorance state*, $s_\top := W \times []$.
- An *initial state* is a subset of the ignorance state.
- The *absurd state* is $\emptyset$.

Notions of *subsistence* and *familiarity* will play extremely important roles in the following discussion, so I'll briefly define them here.

Familiarity, relative to a variable x is a condition that an information state can satisfy (or not) (Heim 1983).

Definition 4.3. Familiarity. A variable x is *familiar* at state s iff $\forall(*, g) \in s[g(x) \neq \star]$

Subsistence captures the idea that two states s and s' can differ only insofar as s' is strictly more anaphorically informative than s, in which $s \prec s'$; *s subsists in s'* (Groenendijk, Stokhof & Veltman 1996).

Definition 4.4. Subsistence A possibility i *subsists* in a state s iff i has a descendent in s:

- $i \prec s \iff \exists i' \in s[i \leq i']$

A state s *subsists* in a state s' iff $s \leq$ is strictly more anaphorically informative than s.

- $s \prec s' \iff s \leq s' \wedge \forall i \in s[i \prec s']$.

4.1.2 Atomic sentences

Given partial assignments, the static semantics of atomic sentences is partial.

Now for the bilateral twist! We recursively define an update function $.[.]^{+,-}$, which given a sentence, returns a function from states to states.

(22) $s[P(x_1, \ldots, x_n)]^+ := \{\, (w, g) \in s \mid [P(x_1, \ldots, x_n)]^{w,g} \text{ is defined and true} \,\}$

(23) $s[P(x_1, \ldots, x_n)]^- := \{\, (w, g) \in s \mid [P(x_1, \ldots, x_n)]^{w,g} \text{ is defined and false} \,\}$

[13]Formally: $s' \geq s \iff \forall i' \in s'[\exists i \in s[i' \geq i]]$.

We can derivatively define the part of s at which ϕ is neither true nor false. This will come in handy later on.

Definition 4.5. The set of possibilities in s at which ϕ is undefined are those which don't subsist in either $s[\phi]^+$ or $s[\phi]^-$.

$s[\phi]^? = \{\, i \in s \mid i \not\prec s[\phi]^{+,-} \,\}$

Note that, unlike in an orthodox update semantics, we're in need of a non-trivial bridge principle to explain how to make sense of assertion.

4.1.3 Negation

Bilateralism will be crucial to validating Double Negation Elimination,[14] which we'll need for both Partee conjunctions and FC with anaphora.

Negative sentences:

(24) $s[\neg\phi]^+ := s[\phi]^-$

(25) $s[\neg\phi]^- := s[\phi]^+$

Fact 4.1. ***DNE:*** $\neg\neg\phi \iff \phi$

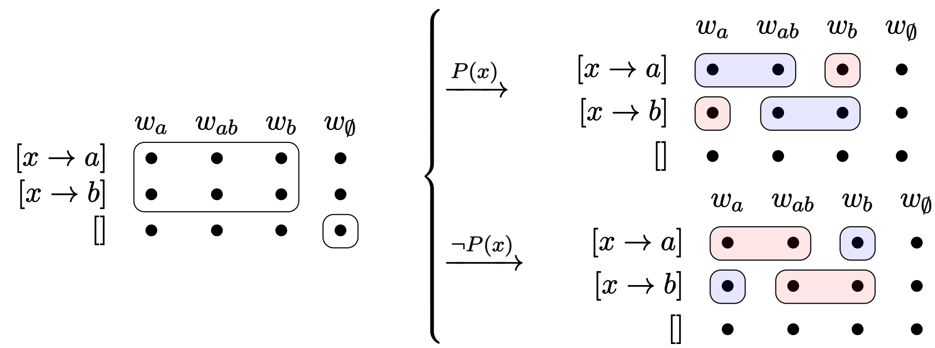
It follows from the definition of negation that (anaphoric) presuppositions project.

Fact 4.2. ***Projection from negative sentences:*** $s[\neg\phi]^? = s[\phi]^?, \forall s$

We're now in a position to appreciate the dynamics of simple sentences. The blue regions reflect the positive update, and the red regions reflect the negative update. The input state on the left is circled.

[14] Just as in (Elliott 2020).

Figure 1: Dynamics of simple sentences.
Subscripts on worlds exhaustively indicate which individuals are P



We'll get to our bridge principle once we discuss how anaphoric information is introduced via existential quantification.

4.1.4 Existential quantification

Existential quantification has a slightly more complex definition in partial update semantics, in order to prevent negated existentials from introducing anaphoric information while validating DNE.

Here we simplify by defining $\exists_x$ in terms of *random assignment* ε_x (i.e., discourse-referent introduction) and successive update.[15]

The negative update crucially just takes the possibilities in s that don't subsist in the positive update, but do subsist in $s[\varepsilon_x][\phi]^-$ (i.e., there should be a way of falsifying).

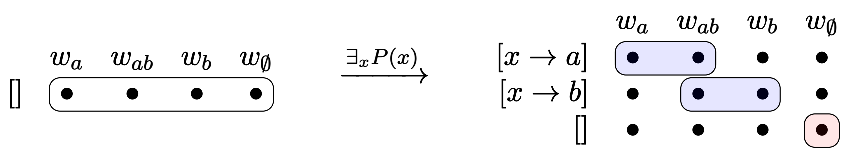
(26) $s[\exists_x \phi]^+ := s[\varepsilon_x][\phi]^+$

(27) $s[\exists_x \phi]^- := \{\, i \in s \mid i \not\prec [\exists_x \phi]^+, i \prec s[\varepsilon_x][\phi]^- \,\}$

[15]Random assignment is defined in the usual fashion as a privileged tautology:

(1) $s[\varepsilon_v] := \{\, (w, h) \in s \mid g[v]h \,\}$

Figure 2: Dynamics of existential statements. Subscripts on worlds exhaustively indicate the individuals that are P.



Given that existential statements can introduce anaphoric information, we can state a couple of useful facts.

Fact 4.3. ***Updates partition (anaphorically more informative) states:*** *for every sentence ϕ, and state s, $s \prec s[\phi]^{+,-,?}$*

Fact 4.4. ***Bivalence.*** *ϕ is bivalent iff for every information state s, $s \prec s[\phi]^{+,-}$.*

This gives rise to a natural notion of assertion and accompanying bridge principle.

Definition 4.6. Assertion. The effect of asserting a sentence ϕ in a context c, if accepted, is $c[\phi]$:

$$c[\phi] = c[\phi]^+ \text{ if } c \prec c[\phi]^{+,-} \text{ else } \emptyset$$

(If ϕ is bivalent then $c[\phi]$ is just $c[\phi]^+$)

What this derives:

- Asserting a sentence with a free variable x at c requires x to be familiar at c (otherwise $c \not\prec c[\phi]^{+,-}$).
- Asserting a negative existential statement won't introduce anaphoric information at c, but will simply narrow down worldly possibilities.
- DNE is valid, so negating a negative existential statement introduces anaphoric information.

(28) John doesn't own a^x car. #It$_x$'s parked outside.

(29) John doesn't own NO^x car. It$_x$'s parked outside!

4.1.5 Logical connectives

We use a technique to lift the Strong Kleene logical connectives into our logical - we interpret each cell in the Strong Kleene truth-table as a successive update (Elliott 2020).

$\phi \vee \psi$	ψ_+	ψ_-	$\psi_?$
ϕ_+	$+$	$+$	$+$
ϕ_-	$+$	$-$	$?$
$\phi_?$	$+$	$?$	$?$

Figure 3: Strong Kleene disjunction

(30) $s[\phi \vee \psi]^+ := s[\phi]^+[\psi]^{+,-,?} \cup s[\phi]^{-,?}[\psi]^+$

(31) $s[\phi \vee \psi]^- := s[\phi]^-[\psi]^-$

This directly accounts for Partee disjunctions. Let's go through a simple example: $\neg\exists_x B(x) \vee U(x)$ "Either there's no bathroom or it's upstairs".

Since $U(x)$ is an atomic sentence, holding the positive update by the first disjunct constant we can simplify.

(32) $s[\neg\exists_x B(x)]^+[U(v)]^{+,-,?} = S[\neg\exists_x B(x)]^+ = \{\, (w,g) \in s \mid I_w(B) = \emptyset \,\}$

Since the first disjunct has bivalent truth-conditions, the second verification case is simply conjunctive by fact 4.1:

(33) $s[\neg\exists_x B(x)]^-[U(v)]^+ = s[\exists_x B(x) \wedge U(x)]^+$

We now take the union of (32) and (33):

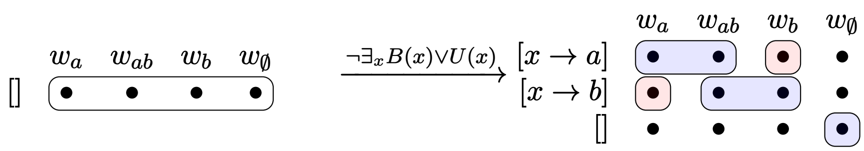
(34) $s[\neg\exists_x B(x) \vee U(x)]^+ = s[\neg\exists_x B(x)]^+ \cup s[\exists_x B(x) \wedge U(x)]^+$

The negative update is easier:

(35) $s[\neg\exists_x B(x) \vee U(x)]^- = s[\exists_x B(x)]^+[U(x)]^- = s[\exists_x B(x) \wedge \neg U(x)]^+$

$\neg\exists_x B(x) \vee U(x)$ is bivalent, since for any information state s, we can partition s into the parts which verify the disjunction, (possibilities where either no bathroom exists, or a bathroom upstairs exists), and the parts which falsify the disjunction (all other possibilities).

Figure 4: Bathrooms exist in all worlds except null subscript; subscripts indicate which of the bathrooms is upstairs.



Note importantly that in the resulting state (the true part), x isn't familiar but is merely *possible* - there are both *bathroom* possibilities, paired with bathroom drefs, and *non-bathroom* possibilities, paired with empty assignments.

This means that a subsequent pronoun won't be licensed, unless the non-bathroom possibilities are contextually eliminated. This prediction is borne out - imagine that *the house being renovated* contextually entails that *there is an upstairs bathroom.*

(36) **A:** Either there's no^x bathroom, or it$_x$'s upstairs. **B:** This house has been recently renovated, so you'll find it$_x$ on the right.

This data is beyond the remit of standard dynamic theories, which assume that disjunctions close off all drefs introduced in their scope (see (Elliott 2020) for an extended version of this argument).

4.2 Epistemic modals in partial update semantics

Epistemic modals are interpreted relative to a *body of information*; in update semantics, this is provided by the input state (Veltman 1996).[16]

Given that we're working in a trivalent system we have to make a decision about how (anaphoric) presuppositions project through epistemic modals.

The basic idea will be that epistemic modals are strong Kleene existential quantifiers over possibilities in the input state.

They impose a weak consistency requirement.

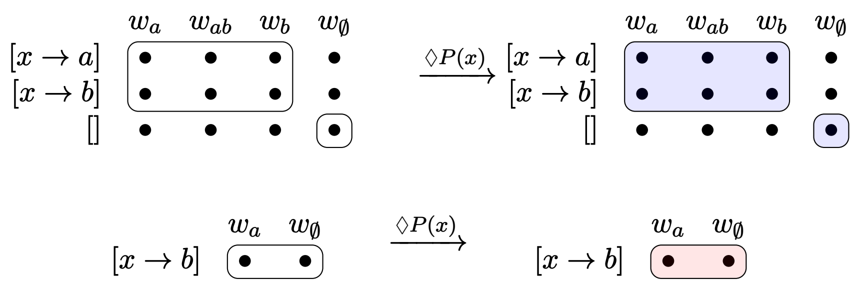
[16] We'll adopt just this view for expositional simplicitly, but see, e.g., (Willer 2018, Goldstein 2019) for a more nuanced view incorporating accessibility relations.

(37) $s[\Diamond\phi]^+ = s$ if $s[\phi]^+ \neq \emptyset$

(38) $s[\Diamond\phi]^- = s$ if $s[\phi]^+ \neq \emptyset \wedge s \prec s[\phi]^-$

The presupposition of $\Diamond\phi$ is essentially disjunctive — either $\exists i \in s$ at which ϕ is true, or $\forall i \in s$ is s.t., ϕ is false.[17]

Figure 5: Updating with possibility statements.



Note that possibility statements have a special status in update semantics, since $s[\Diamond\phi]^+$ and $s[\Diamond\phi]^-$ are mutually exclusive (sentences are true or false throughout the entire state).

This means that we account for discourse such as the following *without* modal subordination, thanks to the weak consistency requirement! Strong inferences in unembedded cases will have to be treated as proviso inferences.

(39) There might be a bathroom. The bathroom might be upstairs.

This gives rise to a natural treatement of $\Box$ as the dual of $\Diamond$:

(40) $s[\Box\phi]^+ = s[\Diamond\neg\phi]^- = s$ if $s \prec s[\phi]^+$ else $\emptyset$

(41) $s[\Box\phi]^- = s[\Diamond\neg\phi]^+ = s$ if $s[\phi]^- \neq \emptyset$

It's easy to see that *must* is strong in partial update semantics - any state that supports $\Box\phi$ will invariably support ϕ, since $\Box\phi$ demands at s that ϕ is true throughout s.[18]

Note that the predictions of test semantics is that modalized statement are anaphorically inert. This doesn't seem right.

[17] A familiar problem - possibility statements are technically uninformative in update semantics. There are various ways of addressing this, many of which cash out the idea that $\Diamond\phi$ "highlights" or "draws attention to" ϕ. I won't discuss this issue here.

[18] see (von Fintel & Gillies 2010) for a defence of strong *must*.

(42) Andreea might have a^v husband. If she's wearing a ring, I'll ask about him$_v$.

(43) There must be a^x bathroom, because I just saw it$_x$!

Instead, we'll suggest that like disjunctions, modalized statements introduce *possible* (i.e., non-familiar) discourse referents. In order to cash this out, we'll implement a relatively small tweak.

4.2.1 Revising $\Diamond$ to account for Partee conjunctions

> Currently, if $s[\Diamond\phi]^+$ is non-empty, it returns s. Instead, if $s[\Diamond\phi]^+$ is non-empty, then it should be strictly anaphorically more information than s - it returns a state that s subsists in.

This is pretty easy in fact, instead of s, we'll take $s[\phi]^{+,-,?}$.

(44) $s[\Diamond\phi]^+ = s[\phi]^{+,-,?}$ if $s[\phi]^+ \neq \emptyset$

(45) $s[\Diamond\phi]^- = s[\phi]^{+,-,?}$ if $s[\phi]^+ \neq \emptyset \wedge s \prec s[\phi]^-$

Figure 6: Standard dynamic theory of epistemic modals.

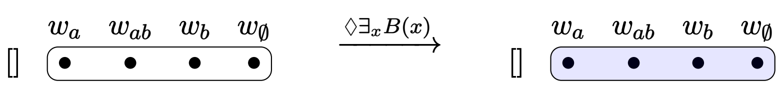
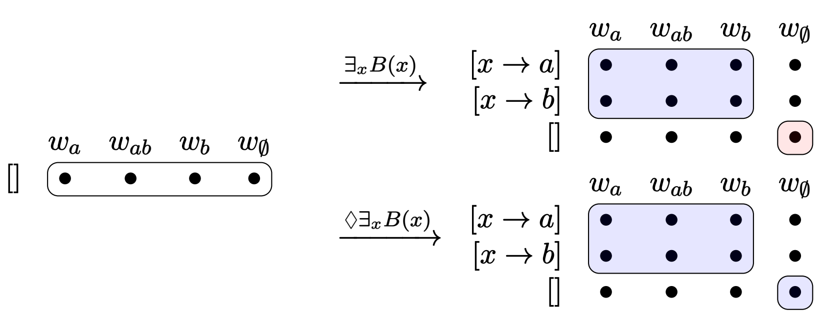


Figure 7: New dynamic theory of epistemic modals.



If the test succeeds, and ϕ is bivalent, then asserting $\Diamond\phi$ is equivalent to asserting $\phi \vee \neg\phi$. $\Diamond\neg\phi$ imposes a different test, but if it succeeds, then asserting $\Diamond\neg\phi$ is equivalent to asserting $\Diamond\phi$, since $\phi \vee \neg\phi \iff \neg\phi \vee \neg\neg\phi$ (by DNE).

This immediately accounts for Partee conjunctions. Since for any sentence the following holds by DNE:

- $s[\phi]^{+,-,?} = s[\neg\phi]^{+,-,?}$

(46) $$s[\neg\exists_x B(x)]^{+,-,?} = s[\exists_x B(x)]^{+,-} = \{\, (w,h) \mid (w,g) \in s, g[x]h \wedge h(x) \in I(B) \,\} \cup \{\, (w,g) \in S \mid I_w(B) = \emptyset \,\}$$

In fact, if $s[\Diamond\neg\exists_x B(x)]^{+} \neq \emptyset$, then $= s[\exists_x B(x)]^{+} \cup s[\neg\exists_x B(x)]$, i.e., $s[\exists_x B(x) \vee \neg\exists_x B(x)]^{+}$

In general, if it's a contextual certaining that ϕ, then $\Diamond\phi$ is infelicitous (presumably for reasons of redundancy).[19]

If it isn't a contextual certainty at s that there is no bathroom, then $\Diamond U(v)$ can have a non-empty positive update, if there are s-possibilities where there is a bathroom upstairs.

(47) Maybe there's no^x bathroom and maybe it$_x$'s upstairs.

(48) $$s[\Diamond\neg\exists_x B(x)]^{+} = s[\exists_x B(x)]^{-} \cup s[\exists_x B(x)]^{+} \text{ if } s[\exists_x B(x)]^{-} \neq \emptyset \text{ else } \emptyset \qquad := s'$$

(49) $$s'[\Diamond U(x)]^{+} = \begin{matrix} s[\exists_x B(x)]^{-} \\ \cup\, s[\exists_x B(x)]^{+} \end{matrix} \text{ if } \begin{matrix} s[\exists_x B(x)]^{-} \neq \emptyset \\ \wedge\, (s[\exists_x B(x)]^{+}[U(x)]^{+} \neq \emptyset \vee s[\exists_x B(x)]^{-}[U(x)]^{+} \neq \emptyset) \end{matrix}$$

As long as the pragmatic constraints on disjunction are satisfied, then modal disjunction will follow, even with anaphora.

It's interesting to note that we account for (50) *without* a distinct mechanism for modal subordination. I suggested that this is a good thing.

(50) Maybe there's a bathroom, and maybe it's upstairs.

This is because, if non-empty, "Maybe there's a bathroom" and "maybe there's no bathroom" introduce the same anaphoric information.

[19] It's a little hard to see exactly how possibility statements could *ever* be redundant in test semantics, since they're never informative (at least, in an informational sense).

5 Incorporating free choice

In order to account for FC with anaphora, we'll incorporate Goldstein's idea that disjunction presupposes homogeneity concerning the modal status of the disjuncts.

For the time being, we'll have to make do with a sketch. The crucial insight is that, now that we have an account of Partee conjunctions the $s[\Diamond\phi \wedge \Diamond\psi]^+ \neq \emptyset$ clause will derive sensible results.

(51) $s[\phi \mathbin{\overline{\vee}} \psi]^+ := s[\phi \vee \psi]^+$ if $s[\Diamond\phi \wedge \Diamond\psi]^+ \neq \emptyset$ else $\emptyset$

(52) $s[\phi \mathbin{\overline{\vee}} \psi]^- := s[\phi \vee \psi]^-$

Note that only an assertion of $\phi \vee \psi$ requires both disjuncts to be possible; $\neg(\phi \vee \psi)$ has a classical meaning - this is important for maintaining an account of *double prohibition*, i.e.:

(53) It's impossible that Jameson is in the park or at the cinema.
  a. *It's impossible that Jameson is at the park*
  b. *It's impossible that Jameson is at the cinema*

5.1 Narrow FC with anaphora

In order to see how we the account works for (narrow) FC with anaphora:

(54) $s[\Diamond(\neg\exists_x B(x) \mathbin{\overline{\vee}} U(x))]^+$

(55) $s[\neg\exists_x B(x) \vee U(x)]^+ = \begin{matrix} s[\neg\exists_x B(x)]^+ \\ \cup\, s[\exists_x B(x) \wedge U(x)]^+ \end{matrix}$ if $s[\Diamond\neg\exists_x B(x) \wedge \Diamond U(x)]^+ \neq \emptyset$ else $\emptyset$

It's obvious that if true, such a context will support $\Diamond\neg\exists_x[B(x)]$ and $\Diamond(\exists_x[B(x) \wedge U(x)])$.[20]

[20]Definitions of support and entailment are the standard dynamic definitions from (Groenendijk, Stokhof & Veltman 1996):

Definition 5.1. Support. s supports ϕ iff $s \prec s[\phi]^+$.

Definition 5.2. Entailment. ϕ entails ψ iff for all information states s, $s[\phi]^+$ supports ψ.

5.2 Future work: wide FC with anaphora

The explanation for wide FC should rely on $\Diamond\Diamond\phi \implies \Diamond\phi$.

(56) Maybe there's no^x bathroom, or maybe it$_x$'s upstairs.

(57) $s[\Diamond\neg\exists_x B(x) \mathbin{\overline{\vee}} \Diamond U(x)]^+$

(58) $s[\Diamond\neg\exists_x B(x) \mathbin{\overline{\vee}} \Diamond U(x)]^+ = s[\Diamond\neg\exists_x B(x)]^+ \cup s[\Box\exists_x B(x) \wedge \Diamond U(x)]^+$ if $s[\Diamond\neg\exists_x B(x) \wedge \Diamond U(x)]^+ \neq \emptyset$

A problem: if the presupposition of the disjunction is satisfied, then the positive extension of the second disjunct is guaranteed to be empty. This seems like a strange result

6 Conclusion

In this talk, we've explored a new spin on the modal-disjunction, connection, suggesting that a possibility statement $\Diamond\phi$, as well as imposing a weak consistency test, assert something like $\phi \vee \neg\phi$.

We used this idea to account for a novel empirical puzzle - *Partee conjunctions* in a principled fashion, and briefly explored a connection with free choice which may help us narrow down the range of viable theories.

One outstanding issue is what we accommodate on the basis of unembedded modal statements. "The bathroom might be upstairs" does seem to require the context to entail the existence of a bathroom.

We actually never run into this issue with anaphora such as "it might be upstairs", since in any case the only way to introduce a possible discourse referent is to assert "there might be a bathroom", so we predict that "it might be upstairs" should be bad in an out-of-the-blue context.

The assertive component of $\Diamond\phi$ is suspiciously similar to the *inquisitive closure* operator $?\phi := \phi \vee \neg\phi$ of inquisitive semantics (Ciardelli, Groenendijk & Roelofsen 2019). One way of thinking about the proposal is that $\Diamond\phi$ dynamically raises the issue *whether* ϕ.

I plan to explore the connection to inquisitive semantics in future work, concretely in order to account for why "the bathroom might be upstairs" seems to have a strong presupposition - the idea is that *might*, as well as raising an issue, is sensitive to issues raised in context.

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