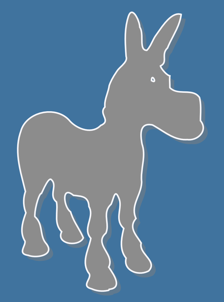

Disjunctive donkeys and overlapping updates

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Background: readings of donkey sentences

First-generation dynamic accounts of anaphora derive ∃-readings for *discourse anaphora* and ∀-readings for *donkey anaphora* as a corollary (Groenendijk & Stokhof 1991, Heim 1982).

(1) Giles owns a donkey and he treasures it. $\exists_d \mathbf{O}(d) \wedge \mathbf{T}(d) \iff \exists_d(\mathbf{O}(d) \wedge \mathbf{T}(d))$

(2) If Giles owns a donkey then he feeds it well. $\exists_d \mathbf{O}(d) \rightarrow \mathbf{T}(d) \iff \forall_d(\mathbf{O}(d) \rightarrow \mathbf{T}d)$

A more nuanced picture: donkey pronouns can also be associated with existential force (Chierchia 1995, Kanazawa 1994); (3) biases a ∃-reading.

(3) I doubt that if Giles owns a donkey he feeds it well.
The speaker believes that if Giles has donkeys, he doesn't feed any of them well

A pertinent distinction that cross-cuts ∀/∃-readings: (2) and (3) have *homogeneous* readings (Champollion, Bumford & Henderson 2019) — in a mixed scenario, where Giles owns a donkey that he feeds well, and a donkey that he doesn't, (2) is false, and the speaker in (3) has false beliefs. *Heterogeneous* readings are also possible, subject to context. (4) can be true in a *mixed scenario*, i.e., if someone with multiple umbrellas left one or more of their umbrellas at home. This is a *heterogeneous* ∃-reading. Heterogeneous ∀-readings are also possible in negative contexts.

(4) Q: *Did anyone get wet?* A: Everyone who has an umbrella remembered to bring it.
Everyone who has at least one umbrella remembered to bring at least one of their umbrellas

Empirical contribution: readings of disjunctive donkey sentences

Disjunctive donkey sentences also give rise to ∃/∀-readings, and thereby allow for both homogeneous/heterogeneous readings depending on the context. Disjunctive donkey sentences are disjunctions where the initial disjunct contains an indefinite and the *negation* of the first disjunct entails a witness to the indefinite (an observation famously attributed to Barbara Partee); this is connected to the possibility of discourse anaphora from under double-negation (6); (5) and (6) are famously outside of the scope of first-generation dynamic theories.

(5) Either there isn't a bathroom in this house, or it's upstairs.

(6) It's not true that there isn't a bathroom in this house. It's upstairs!

What are the truth conditions of disjunctive donkey sentences? Krahmer & Muskens (1995) plausibly claim that (5) has a homogeneous ∀-reading — it's falsified as soon as there is any bathroom that isn't upstairs. Elliott (2023) however shows that, given the right context, a disjunctive donkey sentences can receive a heterogeneous ∃-reading, as in (7). Moreover, *negating* a disjunctive donkey sentence seems to strongly bias a homogeneous ∃-reading; the majority of speakers intuit that the existence of an upstairs bathroom is enough to falsify (8). This parallels (3).

(7) Q: *Did Josie get wet?* A: Either she doesn't own an umbrella, or she brought it with her.
If Josie has an umbrella, she brought at least one her umbrellas

(8) Neither is there no bathroom in this house, nor is it upstairs.

The challenge

We need a theory of ∀/∃-readings that generalizes to disjunctive donkey sentences. To accomplish this, I build on ideas from Champollion, Bumford & Henderson 2019, with a markedly different logical perspective on the core phenomenon based on truth-value *gluts* rather than gaps. The contrast between (9) and (10) illustrates how context can condition the salient reading of a donkey disjunction.

(9) Context: *Spoken by a donkey welfare activist*
If Giles [either has no donkey, or feeds it treats], he's safe from reprisal.
Giles is safe from reprisal only if he feeds all of his donkeys treats

(10) Context: *Spoken by a cruel industrialist*
If Giles [either has no donkey, or feeds it treats], he's working below maximum efficiency.
Giles is working below max efficiency if he feeds any of his donkeys treats

Gluts in the logic of anaphoric dependencies

Homogeneous readings arise when a sentence with anaphora ϕ is interpreted *exhaustively* relative to $\neg\phi$. This is only non-trivial if ϕ and $\neg\phi$ are compatible. A logic of anaphora gives rise to truth-value gluts as soon as the classical equivalence in (11) is validated. This automatically derives a heterogeneous reading for disjunctive donkeys, via some relatively uncontroversial logical principles, together with existential truth-conditions for discourse anaphora.

(11) $\phi \vee \psi \iff \phi \vee (\neg\phi \wedge \psi)$

(12) $\neg\exists_x P(x) \vee Q(x) \Rightarrow \neg\exists_x P(x) \vee (\neg\neg\exists_x P(x) \wedge Q(x))$

(13) $\Rightarrow \neg\exists_x P(x) \vee (\exists_x P(x) \wedge Q(x))$ by Double Negation Elimination

(14) $\Rightarrow \exists_x P(x) \rightarrow (\exists_x (P(x) \wedge Q(x)))$ by Egli's theorem and definition of implication

Now consider the *negation* of a Partee disjunction: de Morgan's and existential readings for discourse anaphora derive a heterogeneous ∃-reading.

(15) $\neg(\neg\exists_x P(x) \vee Q(x))$

(16) $\Rightarrow \neg\neg\exists_x P(x) \wedge \neg Q(x))$ by de Morgan's

(17) $\Rightarrow \exists_x (P(x) \wedge \neg Q(x))$ by Double Negation Elimination and Egli's theorem

Important: the donkey disjunction (14) and its negation (17) are *compatible* — both are true in a *mixed* scenario, i.e., where $P \cap Q \neq \varnothing$ and $P - Q \neq \varnothing$. By combining (11) with some straightforward logical principles we've arrived at a truth-value *glut*. Interpreting (14)/(17) exhaustively gives rise to a homogeneous reading:

(18) $\exists_x P(x) \rightarrow (\exists_x (P(x) \wedge Q(x))) \wedge \overbrace{\neg\exists_x (P(x) \wedge \neg Q(x))}^{\text{exhaustive inference}}$
$\Rightarrow \exists_x P(x) \rightarrow \forall_x (P(x) \rightarrow Q(x))$ *If there is a P then every P is Q*

(19) $\exists_x (P(x) \wedge \neg Q(x)) \wedge \neg(\exists_x P(x) \rightarrow (\exists_x (P(x) \wedge Q(x))))$
$\Rightarrow \exists_x P(x) \wedge \forall_x (P(x) \rightarrow \neg Q(x))$ *There is a P and no P is Q*

Bilateral Update Semantics

The task now is to develop a dynamic semantics that can capture donkey disjunctions and derives the logical results described above. To do so, I'll present a bilateral extension of of Heimian update semantics (Heim 1982, Veltman 1996, Groenendijk, Stokhof & Veltman 1996) using the strong Kleene interpretation schema (Elliott 2022).

(20) **Atomic sentences**: $s[\text{it}_x \text{ is upstairs}]^+ := \{ (w,g) \in s \mid g_x \text{ is upstairs in } w \}$
$s[\text{it}_x \text{ is upstairs}]^- := \{ (w,g) \in s \mid g_x \text{ isn't upstairs in } w \}$

(21) **Existentials:** $s[\text{there is a}^x \text{ bathroom}]^+ := \{ (w,h) \mid (w,g) \in s, g[x]h \wedge h_x \text{is a bathroom in } w \}$
$s[\text{there is a}^x \text{ bathroom}]^- := \{ (w,g) \in s \mid \text{there is no bathroom in } w \}$

(22) **(Flip-flop) negation:** $s[\text{not } \phi]^+ := s[\phi]^-$ $\quad s[\text{not } \phi]^- := s[\phi]^+$

Consequence of flip-flop negation: DNE is valid; $s[\text{there's no}^x \text{ bathroom}]^-$ introduces a bathroom discourse referent. The updates for disjunction are computed on the basis of the strong Kleene truth table for disjunction; the *unknown update* plays the role of the third truth value in a trivalent semantics.

(23) **The unknown update:** $s[\phi]^? = \{ i \in s \mid i \not< s[\phi]^{+,-} \}$

(24) **Disjunction:** $s[\phi \vee \psi]^+ := s[\phi]^+[\psi]^+ \cup s[\phi]^+[\psi]^- \cup s[\phi]^+[\psi]^? \cup s[\phi]^-[\psi]^+ \cup s[\phi]^?[\psi]^+$
$s[\phi \vee \psi]^- := s[\phi]^-[\psi]^-$

Applying the disjunctive schema to "Either there is no bathroom or its upstairs".

(25) **Bathroom disjunction: postive update**

a. $s[\text{there is no bathroom}]^-[\text{it's upstairs}]^+ = s[\text{there's a bathroom}]^+[\text{it's upstairs}]^+$

b. $s[\text{there is no bathroom or it's upstairs}]^+ = \{ (w,g) \in s \mid \text{no bathroom in } w \}$
$\cup \left\{ (w,h) \;\middle|\; \begin{array}{l} (w,g) \in s, g[x]h, \\ h_x \text{ an upstairs bathroom in } w \end{array} \right\}$

(26) **Bathroom disjunction: negative update**
$s[\neg\exists_x B(x) \vee U(x)]^- = \{ (w,h) \mid (w,g) \in s, g[x]h \wedge h_x \text{ is a non-upstairs bathroom in } w \}$

Weak assertion derives heterogeneous readings

If we assume a natural bridge principle for BUS, then we'll automatically derive heterogeneous readings for disjunctive donkeys and their negated counterparts. Note that the definedness clause imposes Heimian familiarity. *Mixed* possibilities (i.e., those with a bathroom upstairs and downstairs) will survive update, since all that matters is that they subsist in the positive update.

(27) **Assertion (first attempt):** $c + \phi = c[\phi]^+$, defined only if $c[\phi]^? = \varnothing$

Homogeneous readings via exhaustive interpretation

How do we derive the (often more salient) homogeneous reading? Note that $s[\phi]^+$ and $s[\phi]^-$ may overlap, which corresponds to a truth-value *glut*. The key idea is to derive homogeneous readings via *exhaustive* interpretation (Groenendijk & Stokhof 1984); ϕ is interpreted exhaustively wrt the question induced by $\{\phi, \neg\phi\}$. For a donkey disjunction $\neg\exists_x P(x) \vee Q(x)$, the relevant alternatives correspond to the formulas in (28), which induce a partition with three cells:

(28) $\{$ **pos alt:** $\exists_x P(x) \rightarrow \exists_x (P(x) \wedge Q(x))$, **neg alt:** $\exists_x (P(x) \wedge \neg Q(x)) \}$

(29)
- **Cell 1**: if there's a P, then every P is Q — *pos alt true; neg alt false*
- **Cell 2**: there's a P and no P is Q — *pos alt false; neg alt true*
- **Cell 3**: there's a P that's a Q, and a P that's not a Q — *both alts false*

(30) **Assertion (exhaustive):** $c + \phi = \{ (w,g) \mid (w,g) \in c[\phi]^+, (w,*) \notin c[\phi]^- \}$,
defined only if $c[\phi]^? = \varnothing$

Heterogeneous readings and the QuD

We observed that heterogeneous readings *are* however also possible, subject to contextual factors. Here I follow Champollion, Bumford & Henderson's (2019) elegant idea of relativizing interpretation to a QuD (i.e., a contextually salient equivalence relation $\sim_Q$). The exhaustive bridge principle is correspondingly weakened:

(31) **Assertion (Q-relative):** $c + \phi = \{ (w,g) \in c[\phi]^+ \mid \exists w', w \sim_Q w', (w',*) \notin c[\phi]^- \}$,
defined only if $c[\phi]^? = \varnothing$

For example, imagine the QuD is *Did Josie get wet?*; the context-set is partitioned into *Josie-gets-wet* possibilities, and *Josie-stays-dry* possibilities. Crucially, Josie having at least one umbrella with her contextually entails that she stays dry. The salient question therefore elides any distinction between worlds in which Josie has all of her umbrellas with her, and worlds in which she left some of her umbrellas at home, and therefore "Either Josie doesn't have an umbrella, or she brought it with her" can receive a heterogeneous ∃-reading, in spite of exhaustivity.

Open issue: ∀-readings of discourse anaphora?

A logic which validates existential readings of discourse anaphora *and* de Morgan's predicts gluts more generally, even for conjunctive sentences — it follows that we might expect homogeneous ∀-readings of discourse anaphora in cases like *Q: did Josie get wet?* "Josie has an umbrella but she left it at home" (Chatain 2018).

(32) $\exists_x P(x) \wedge Q(x) \Rightarrow \exists_x (P(x) \wedge (Q(x)))$ *There is a P that's Q*

(33) $\neg(\exists_x P(x) \wedge Q(x)) \Rightarrow \exists_x P(x) \rightarrow \exists_x (P(x) \wedge \neg Q(x))$ *If there's a P, there's a P that's not a Q*

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