Mixed-polarity pluralities: A solution to van Benthem's problem

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Cumulative readings with modified numerals

Modified numerals such as "exactly two" are plural expressions, and can give rise to **cumulative readings**, alongside (doubly) distributive readings.

(1) Exactly two boys ate exactly two pizza slices.

- **Cumulative reading:** $\#\{x \in \textbf{boy} \mid x \text{ ate a slice}\} = 2, \#\{y \in \textbf{slice} \mid y \text{ eaten by a boy}\} = 2$
- Distributive reading: $\#\{x \in \textbf{boy} \mid \#\{y \in \textbf{slice} \mid y \text{ eaten by } x\} = 2\} = 2$

The possibility of cumulative readings suggests that "exactly two boys" existentially quantifies over pluralities, but we still want to encode the upper-bound. This can be accomplished by adding maximality:

(2) Problematic semantics for "exactly two boys":
$$\lambda P\,.\,\exists X\begin{bmatrix}{}^*\textbf{boy}(X) \wedge \# = 2 \wedge P(X)\\ \wedge \neg\exists X'[{}^*\textbf{boy}(X') \wedge X < X' \wedge P(X')]\end{bmatrix}$$

This works for simple distributive sentences such as "exactly two boys sneezed" but fails to capture the attested cumulative reading of (1) (Brasoveanu 2013, Charlow 2016), instead predicting (3). Concretely, (3) holds in a scenario where three boys a, b, c each ate a distinct slice, since e.g., a, b between them ate two slices, and no larger group between them ate two slices. Intuitively, the cumulative reading of (1) is false in this scenario.

(3) Pseudo-cumulative truth-conditions for (1):
$\exists X, \#X = 2, X$ are boys who between them ate ex. 2 slices
No boy-plurality larger than X between them ate ex. 2 slices.

My goal: I show that van Benthem's problem can be avoided by adopting a non-standard ontology for pluralities, while maintaining an austere semantics for numerals. Concretely, I'll exploit an idea due to Bledin (2024) that the domain of individuals can encode both positive and negative information.

Polarizing the domain

Bledin's innovation: a 'polarized' domain $D^{\pm}$ encoding a distinction between *positive* and *negative* individuals. Each individual $x \in D$ is associated with a unique 'negative counterpart' x^- (pronounced "not x"), as well as a 'positive counterpart' x^+, which stands in for x itself. There is a one-to-one relationship between individuals and their positive/negative counterparts.

(4) $D := \{a, b, c, \ldots\} \qquad D^{\pm} := \{a^+, a^-, b^+, b^-, c^+, c^-, \ldots\}$

Core intuition behind $D^{\pm}$: x^+ indicates x's participation in some yet-to-be-named event, and x^- marks x's non-participation.

$D^{\pm}$ is *closed under sum-formation* (Link 1983), with an important proviso: 'incoherent' sums are removed (Akiba 2009).

- A sum $X \in D^{\pm}$ is incoherent if $\exists x \in D, x^+ \leq_{At} X \wedge x^- \leq_{At} X$.

As a result, $D^{\pm}$ contains many different kinds of sums, e.g.

- Wholly positive sums, e.g., $a^+ \oplus b^+$.
- Wholly negative sums, e.g., $a^- \oplus b^-$.
- Mixed-polarity sums, e.g., $a^+ \oplus b^-$, and $a^- \oplus b^+$

I assume that distributive predicates are true/false of individuals in D. Composition of $X \in D^{\pm}$ is mediated via the *distributivity operator* Δ, which, as usual, universally quantifies over atoms. For each x^+ it asserts that x is true of P, and for each x^-, that x is false of P (I ignore homogeneity here).

(5) $\Delta(P) := \lambda X \in D^{\pm}\,.\,\forall x^+ \leq_{At} X, P(x) = 1,$
$\qquad\qquad\qquad \forall x^- \leq_{At} X, P(x) = 0$

(6) $\Delta(\textbf{swim})(a^+ \oplus b^- \oplus c^+) \iff a, c$ swim and b doesn't swim

Plurality and maximality

I depart from Bledin 2024 in assuming that plural noun denotations such as ***boy** range over **maximal** pluralities $D^{\pm}$. This means that for every $x \in$ **boy**, $X \in {}^*$**boy** contain either x^+ or x^- as an atomic part. Due to the ban on incoherent sums, there are potentially many such maximal pluralities. The semantics for plural marking is given in (7).

(7) ${}^*P := \textbf{Max}_{\leq}\{X \in D^{\pm} \mid \forall x^{\pm} \leq_{At} X, P(x)\}$

(8) assume **boy** $:= \{a, b, c\}$, then:
$${}^*\textbf{boys} = \left\{\begin{array}{c} a^+ \oplus b^+ \oplus c^+ \\ a^+ \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^+, a^- \oplus b^+ \oplus c^+ \\ a^- \oplus b^- \oplus c^+, a^- \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^-, \\ a^- \oplus b^- \oplus c^- \end{array}\right\}$$

It's perhaps helpful to think of pluralities in ***boy** as specifying, for every boy, whether or not he participated in some yet-to-be-specified event, i.e.,

- $a^+ \oplus b^+ \oplus c^- \rightsquigarrow$ out of the boys, only a and b did a particular thing

Determiners as predicates of pluralities

Conventions for defining (plural) determiners:

(9) Ordinary individuals with pos/neg counterparts in X:
a. $X^+ := \{x \mid x^+ \leq_{At} X\}$
b. $X^- := \{x \mid x^- \leq_{At} X\}$

- $(a^+ \oplus b^+ \oplus c^-)^+ = \{a, b\}$
- $(a^+ \oplus b^+ \oplus c^-)^- = \{c\}$

All (and only) the conservative determiners can be defined as predicates of pluralities by placing constraints on X^+ and X^- (come to my talk at the *Too many tools* workshop for more on this).

(10) $[\![\text{some}]\!] = \{X \in D^{\pm} \mid X^+ \neq \varnothing\}$
(11) $[\![\text{all}]\!] = \{X \in D^{\pm} \mid X^- = \varnothing\}$
(12) $[\![\text{no}]\!] = \{X \in D^{\pm} \mid X^+ = \varnothing\}$
(13) $[\![\text{not all}]\!] = \{X \in D^{\pm} \mid X^- \neq \varnothing\}$

Plural determiners compose with predicates via intersective modification; DPs come to denote sets of pluralities. Assume **boy** $:= \{a, b, c\}$:

(14) $[\![\text{some}]\!] \cap {}^*\textbf{boy}$
$$= \left\{\begin{array}{c} a^+ \oplus b^+ \oplus c^+, \\ a^+ \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^+, a^- \oplus b^+ \oplus c^+ \\ a^- \oplus b^- \oplus c^+, a^- \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^-, \end{array}\right\}$$

(15) $[\![\text{no}]\!] \cap {}^*\textbf{boy} = \{\, a^- \oplus b^- \oplus c^-, \,\}$

The resulting set is type-lifted via *existential raising* (*ER*) (Winter 2001), generalizing a strategy for composing numerals.

(16) $ER := \lambda Q\,.\,\lambda P\,.\,\exists X \in Q, P(X) = 1$

(17) Some boys swim.
$ER([\![\text{some}]\!] \cap {}^*\textbf{boy})(\Delta(\textbf{swim})(X))$
$\Rightarrow a, b, c$ swim $\vee$ a, b and not c swim $\vee$ a, c and not b swim, ...

(18) No boys swim.
$ER([\![\text{no}]\!] \cap {}^*\textbf{boy})(\Delta(\textbf{swim})(X))$
$\Rightarrow \Delta(\textbf{swim})(a^- \oplus b^- \oplus c^-)$
$\Rightarrow$ a doesn't swim, b doesn't swim, and c doesn't swim

Numeral semantics

Numerals place cardinality constraints on X^+.

- Because maximality is inherent to the plural NP, bare numerals are distinguished from upper-bounded numerals by having an 'at least' semantics (cf. Winter 2001)
- The semantics for "less than n" avoids the existential entailment problem noted by Buccola & Spector 2016.

(19) $[\![\text{two}]\!] = \{X \in D^{\pm} \mid \#X^+ \geq 2\}$
(20) $[\![\text{exactly two}]\!] = \{X \in D^{\pm} \mid \#X^+ = 2\}$
(21) $[\![\text{less than 3}]\!] = \{X \in D^{\pm} \mid \#X^+ < 2\}$
(22) $[\![\text{between 3 and 5}]\!] = \{X \in D^{\pm} \mid 3 \leq \#X^+ \leq 5\}$

(23) $[\![\text{two}]\!] \cap {}^*\textbf{boy} = \left\{\begin{array}{c} a^+ \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^+, \\ a^- \oplus b^+ \oplus c^+, a^+ \oplus b^+ \oplus c^+ \end{array}\right\}$

(24) $[\![\text{ex. two}]\!] \cap {}^*\textbf{boy} = \left\{\begin{array}{c} a^+ \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^+, \\ a^- \oplus b^+ \oplus c^+ \end{array}\right\}$

ER together (24) correctly derives upper-bounded truth-conditions, resolving one component of van Benthem's problem.

(25) Exactly two boys swim.
$ER([\![\text{exactly two}]\!] \cap {}^*\textbf{boy})(\Delta(\textbf{swim})(X))$
$\Rightarrow a, b$ swim and c doesn't or a, c swim and b doesn't or b, c swim and a doesn't
$\Rightarrow \#\{x \in \textbf{boy} \mid x \text{ swims}\} = 2$

(26) Two boys swim.
$ER([\![\text{two}]\!] \cap {}^*\textbf{boy})(\Delta(\textbf{swim})(X))$
$\Rightarrow a, b$ swim and c doesn't or a, c swim and b doesn't or b, c swim and a doesn't or a, b, c all swim
$\Rightarrow \#\{x \in \textbf{boy} \mid x \text{ swims}\} \geq 2$

Deriving cumulative readings

The distributivity operator Δ introduced a sensitivity to positive vs. negative information. The same strategy can easily be extended to Beck & Sauerland's (2000) *cumulation* operator $.^{**}$, which they define as follows:

(27) Beck & Sauerland's operator:
$R^{**}(X, Y) \iff \forall x \in X, \exists y \in Y, R(x, y)$
$\qquad\qquad\qquad \wedge \forall y \in Y, \exists x \in X, R(x, y)$

The intuition for *polarized* pluralities: $X, Y \in D^{\pm}$ are cumulatively true of R just in case $R^{**}(\oplus X^+, \oplus Y^+)$ (given Beck & Sauerland's definition), and $R(x, y)$ doesn't hold for any $x \in X^-$, or any $y \in Y^-$.

Deriving cumulative readings cont

(28) The **polarized** cumulation operator:
$\forall X, Y \in D^{\pm}, R^{**}(X, Y)$
$\iff R^{**}(\oplus X^+, \oplus Y^+) \wedge \forall x \in X^-, \neg\exists y \in Y^+ \cup Y^-, R(x, y)$
$\qquad\qquad\qquad \wedge \forall y \in Y^-, \neg\exists x \in X^+ \cup X^-, R(x, y)$
Roughly: *All (and only) the positive parts of X, Y participated in R.*

(29) Exactly two boys ate exactly two pizza slices.
$\exists X \in ([\![\text{ex. 2}]\!] \cap {}^*\textbf{boy}), \exists Y \in ([\![\text{ex. 2}]\!] \cap {}^*\textbf{slice}), \textbf{eat}^{**}(X, Y))$

(30) $\exists X \in \left\{\begin{array}{c} a^+ \oplus b^+ \oplus c^-, \\ a^+ \oplus b^- \oplus c^+, \\ a^- \oplus b^+ \oplus c^+ \end{array}\right\}, Y \in \left\{\begin{array}{c} s_1^+ \oplus s_2^+ \oplus s_3^-, \\ s_1^+ \oplus s_2^- \oplus s_3^+, \\ s_1^- \oplus s_2^+ \oplus s_3^+ \end{array}\right\}, \textbf{eat}^{**}(X, Y)$
a. $\Rightarrow \textbf{eat}^{**}(a \oplus b)(s_1 \oplus s_2)$
$\wedge \neg\exists x \in \textbf{slice}, \textbf{eat}(c, x) \wedge \neg\exists y \in \textbf{boy}, \textbf{eat}(y, s_3)$
or $\textbf{eat}^{**}(a \oplus c)(s_1 \oplus s_3)$
$\wedge \neg\exists x \in \textbf{slice}, \textbf{eat}(b, x) \wedge \neg\exists y \in \textbf{boy}, \textbf{eat}(y, s_3)$,
... etc.
b. $\Rightarrow \#\{y \in \textbf{boy} \mid y \text{ ate a slice}\} = 2$
$\wedge \#\{x \in \textbf{slice} \mid y \text{ eaten by a boy}\} = 2$

Bonus: cumulative readings with *zero*

Unlike in a standard setting, *zero* can be treated in a way parallel to other numerals (with the proviso that it must have an *exactly* semantics to avoid triviality; Bylinina & Nouwen 2018).

(31) $[\![\text{zero}]\!] = \{X \in D^{\pm} \mid \#X^+ = 0\}$

Novel empirical claim: zero allows for cumulative readings:

(32) *We're tallying up how many people ate what at the joint linguistics-philosophy dinner. I thought that philosophers loved pizza, but...*
Zero philosophers ate zero slices of pizza.

Given **philosopher** $:= \{p, q, r\}$, and **slice** $:= \{s_1, s_2, s_3\}$:

(33) a. $[\![\text{zero}]\!] \cap {}^*\textbf{philosopher} = \{p^- \oplus q^- \oplus r^-\}$
b. $[\![\text{zero}]\!] \cap {}^*\textbf{slice} = \{s_1^- \oplus s_2^- \oplus s_3^-\}$

(34) $\textbf{eat}^{**}(p^- \oplus q^- \oplus r^-, s_1^- \oplus s_2^- \oplus s_3^-)$
$\Rightarrow$ *No philosopher ate any slice*

Bylinina & Nouwen's (2018) semantics for "zero" extends the lattice-theoretic approach to plurality with a bottom element $\bot$: "zero boys swam" is true iff the *maximal* plurality of boys X that swam is s.t., $\#X = 0$; this plurality will be $\bot$ just in case no boys swam, and $\#\bot = 0$. An argument for negative individuals: presupposition projection from the scope of *zero* (Filipe Hisao Kobayashi, p.c.).

(35) Zero boys stopped smoking.
presupposes: *every boy used to smoke*

The basic idea: each individual with a positive/negative counterpart in the DP extension must satisfy the presuppositions of the scope; if the DP denotes $\{a^- \oplus b^- \oplus c^-\}$, then each of a, b, c must be have smoked, in order for the sentence to be defined. On Bylinina & Nouwen's account, $\bot$ doesn't encode any information about the NP restrictor.

Conclusion and outlook

There are of course other approaches to van Benthem's problem; many exploit heavy-duty semantic machinery such as post-suppositions (Brasoveanu 2013), context updates (Charlow 2016), or multi-dimensionality (Haslinger & Schmitt 2020) in order to divorce the scope of the maximality condition in (3) from the numeral. Once positive/negative counterparts are introduced, no such machinery is needed — maximality is encoded directly in plural NP denotations.

An explicit comparison with existing approaches is left to future work. Additionally, open questions remain:

- **Semantic singularity:** In order to give a uniform semantics for determiners, *singular* NPs should also denote elements of $D^{\pm}$; what exactly does semantic singularity contribute?
- **Collective predication:** collective predicates are not integrated in this fragment. Do we need negative groups, in addition to negative atoms?
- Other potential applications, such as homogeneity, exceptive constructions, and negation in collective conjunction (see especially Bledin 2024).

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