Restricting determiners

Conservativity and negative counterparts

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Does semantics have a 'too many tools' problem? ~ September 20, 2024, Noto

- The simple, but powerful tools commonly assumed in formal semantics, e.g., arbitrary functional types and higher-order functions, leads to an expressivity problem.
- A particular manifestation of this problem: a broad class of universally unattested non-conservative determiners can easily be expressed as higher-order functions.
- My approach:
  - Perhaps higher-order functions are the wrong tool.
  - Expanding the set of possible individuals will allow determiner meanings to be recast as *predicates of pluralities*.

Roadmap

- Background:
    - Determiner meanings in GQ-theory.
    - Conservativity.
    - Warming up with numeral semantics.
- Negative individuals.
    - Introducing the main formal innovation.
    - Incorporating plurality and maximality.
- Application to numerals.
- Extension to other determiners.
    - The non-expressibility of non-conservative determiners.
    - Any conservative determiner is expressible.

Background

- A determiner-meaning in GQ-theory is modeled as a binary relation between sets of individuals A, B (Barwise & Cooper 1981, Keenan & Stavi 1986):
  - A: the restrictor.
  - B: the scope.

(1) a. $\textbf{some}(A, B) \iff A \cap B \neq \varnothing$

b. $\textbf{every}(A, B) \iff A \subseteq B$

c. $\textbf{exactly three}(A, B) \iff \#(A \cap B) = 3$

d. $\textbf{most}(A, B) \iff \#(A \cap B) > \#(A - B)$

- A cherished semantic universal: all attested determiner-meanings in natural language are *conservative*.

(2) An NL determiner *Det* is conservative iff:
$Det(A, B) \iff Det(A, A \cap B)$

- A corollary: $B - A$ may not effect the truth of $Det(A, B)$, if *Det* is conservative.

- The conservativity universal is substantive; non-conservative determiners are easily expressible, e.g., the Härtig quantifier I.

(3) $I(A, B) \iff \#A = \#B$

- Assume
  - $A = \{a\}; \#A = 1$
  - $B = \{b\}; \#B = 1$
  - $A \cap B = \varnothing; \#(A \cap B) = 0$
- $\#A = \#B$
- $\#A \neq \#(A \cap B)$

- The ‘textbook’ treatment of determiners in compositional semantics integrates them as higher-order functions via currying (Heim & Kratzer 1998).

$$[\![\text{some}]\!] := \lambda A \in D_{\langle e,t\rangle} . \lambda B \in D_{\langle e,t\rangle} . \mathbf{some}\begin{pmatrix} \{ x \in D \mid A(x) = 1 \}, \\ \{ x \in D \mid B(x) = 1 \} \end{pmatrix}$$

- Such meanings are easily integrated into the compositional regime thanks to arbitrary functional types.
  - This leads to an *expressivity* problem, since lexical entries for non-conservative determiners can easily be stated.
  - Nevertheless, the GQ-theoretic approach is the de facto standard in formal semantics.

- There's an alternative to GQ-theory, developed specifically for bare numerals.
    - Numerals are decomposed into cardinality predicates + covert existential quantification over pluralities (Link 1987, Verkuyl 1993, Carpenter 1998).

(4) Three boys sneezed.
$\exists X$, *X is a plurality of boys*, $\#X = 3$, *each of X sneezed.*

- Ingredients (Winter 2001):
    - Numerals as predicates of pluralities (in the sense of Link 1983).
    - *ER*: Existential Raising.
    - Δ: The distributivity operator.

(5) a. $[\![\text{three}]\!] = \lambda X \,.\, \#X = 3$

b. $\Delta(P) := \lambda X \,.\, \forall x \leq_{At} X, P(X)$

c. $ER(Q) := \lambda P \,.\, \exists X[Q(X) \wedge P(X)]$

(6) Three boys sneezed.

$ER(\lambda X \,.\, [\![\text{three}]\!]\,(X) \wedge [\![\text{boys}]\!]\,(X))(\Delta([\![\text{sneezed}]\!]))$

$\Rightarrow \exists X[\#X = 3, {}^{*}\mathbf{boy}(X), \forall x \leq_{At} X[\mathbf{sneezed}(x)]]$

- Resulting truth-conditions equivalent to those resulting from the GQ-theoretic determiner **three**.
- Other determiners cannot be reanalyzed in this way, given standard assumptions.

Roadmap

- Goal: a compositional regime for (plural) determiners, in which non-conservative meanings are not expressible.
- Basic ingredients:
  - Existential raising.
  - Distributivity.
  - Determiners as predicates.
- Making sense of determiners-as-predicates will require a re-jigging of the role of *individuals* in semantics.
- Concretely, I'll exploit an idea due to Bledin (2024) that the domain of individuals encodes a distinction between positive and negative information.

Negative individuals

Polarizing the domain

- Main innovation of Bledin (2024): the move from a domain of ordinary individuals to a polarized domain (see also Akiba 2009).
- The polarized domain $D^{\pm}$ contains, for each individual $x \in D$:
  - x^{+}: x's *positive counterpart*.
  - x^{-}: x's *negative counterpart* (pronounced "not x").

$$D := \{ a, b, c \}$$

$$D^{\pm} = \{ a^{+}, a^{-}, b^{+}, b^{-}, c^{+}, c^{-}, \ldots \}$$

- Ordinary individuals are in a one-to-one relationship with their positive/negative counterparts.

- Negative individuals can be thought of as a formal device for encoding an individual's non-participation.
  - If Jimmy happens to be swimming, then Jimmy^{-} is not swimming, and if Jimmy is not swimming, then Jimmy^{-} is swimming.

- I'll model pluralities as i-sums (Link 1983).
- The polarized domain $D^{\pm}$ is constructed in three steps:
  - Take the smallest set containing x^+ and x^-, for every individual $x \in D$.
  - Close the resulting set under sum-formation $\oplus$.
  - Remove incoherent pluralities (Akiba 2009).

(7) A plurality X is incoherent, if there is some $x \in D$, s.t., $x^+ \leq_{At} X$ and $x^- \leq_{At} X$

- Importantly, this means that $D^{\pm}$ is *not closed under* $\oplus$.
  - $a^+ \oplus b^+$ is coherent.
  - $a^+ \oplus b^+ \oplus b^-$ is incoherent.
- The resulting structure is a sub-lattice with multiple maximal elements, given a base domain with multiple elements.

Plurality cont.

$$D := \{a, b, c\}$$

$$D^{\pm} := \left\{ \begin{array}{c} a^+ \oplus b^+ \oplus c^+, \\ a^+ \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^+, a^- \oplus b^+ \oplus c^+, \\ a^- \oplus b^- \oplus c^+, a^- \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^-, \\ a^- \oplus b^- \oplus c^-, \\ a^+ \oplus b^+, a^+ \oplus b^-, a^- \oplus b^+, a^- \oplus b^-, \\ a^+ \oplus c^+, a^+ \oplus c^-, a^- \oplus c^+, a^- \oplus c^-, \\ b^+ \oplus c^+, b^+ \oplus c^-, b^- \oplus c^+, b^- \oplus c^-, \\ a^+, a^-, b^+, b^-, c^+, c^- \end{array} \right\}$$

- The resulting plural polarized domain, which from here on we'll refer to as $D^{\pm}$, thus contains many different pluralities, alongside positive/negative atoms:
  - Wholly-positive pluralities, e.g., $a^+ \oplus b^+$; "a and b"
  - Wholly-negative pluralities, e.g., $a^- \oplus b^-$; "not a and not b"
  - Mixed-polarity pluralities, e.g., $a^+ \oplus b^-$; "a and not b"
  - A useful convention when talking about pluralities in the polarized domain:
    - $X^+ = \{ x \in D \mid x^+ \leq_{At} X \}$
    - $X^- = \{ x \in D \mid x^- \leq_{At} X \}$
- E.g.,:
  - $(a^+ \oplus b^-)^+ = \{ a \}$
  - $(a^+ \oplus b^-)^- = \{ b \}$

- I'll assume that distributive predicates are still true of ordinary individuals.
- Composition with elements of $D^{\pm}$ is mediated by the distributivity operator Δ, which has the following definition (ignoring homogeneity):

(8) Polarized distributivity operator:

$$\Delta(P) := \lambda X \in D^{\pm} . \forall x \in X^{+}, P(x) = 1 \\ \wedge \forall x' \in X^{-}, P(x') = 0$$

- $\Delta(\textbf{swim})(a^{+} \oplus b^{+} \oplus c^{-}) \iff a, b$ both swim and b doesn't swim

- How do NPs come to introduce elements of $D^{\pm}$?
- I'll assume that the contribution of plural marking is to take the maximal elements of $D^{\pm}$, such that every atomic part is the pos/neg counterpart of an individual with the NP-property.

(9) $[\![\text{boy}]\!] = \{a, b, c\}$

(10) $$[\![\text{boys}]\!] = \textbf{Max}_{\leq}\left\{ X \in D^{\pm} \,\middle|\, \forall x \in X^{+} \cup X^{-}, [\![\text{boy}]\!](x) \right\}$$
$$= \left\{ \begin{array}{c} a^{+} \oplus b^{+} \oplus c^{+}, \\ a^{+} \oplus b^{+} \oplus c^{-}, a^{+} \oplus b^{-} \oplus c^{+}, a^{-} \oplus b^{+} \oplus c^{+}, \\ a^{-} \oplus b^{-} \oplus c^{+}, a^{-} \oplus b^{+} \oplus c^{-}, a^{+} \oplus b^{-} \oplus c^{-}, \\ a^{-} \oplus b^{-} \oplus c^{-} \end{array} \right\}$$

- A useful isomorphism: elements of ⟦boys⟧ express total mappings from boys to truth-values, depending on whether he participated in some yet-to-be-named eventuality (Amir Anvari, p.c.).

$$a^+ \oplus b^+ \oplus c^- \approx \begin{bmatrix} a \rightarrow 1 \\ b \rightarrow 1 \\ c \rightarrow 0 \end{bmatrix}$$

- More generally, elements of $D^{\pm}$ are isomorphic to partial functions from D to $\{1, 0\}$.
  - I'll come back to this correspondence later.

Application to numerals

- We can reconstruct a semantics for numerals as predicates of elements of $D^{\pm}$.
- Idea: numerals place cardinality constraints on the number of individuals with positive counterparts in a plurality.
- Importantly, since maximality is inherent in plural marking, numerals must have an *at least* semantics (cf. Winter 2001).

(11) $\textbf{two} := \{ X \in D^{\pm} \mid \#X^{+} \geq 2 \}$

(12) $\textbf{two} := \{X \in D^{\pm} \mid \#X^{+} \geq 2\}$

(13) $\textbf{two} \cap [\![\text{boys}]\!]$

$$= \textbf{Max}_{\leq}\{X \in D^{\pm} \mid \#X^{+} \geq 2, \forall x \in X^{+} \cup X^{-}, \textbf{boy}(x)\}$$

$$= \left\{ \begin{array}{c} a^{+} \oplus b^{+} \oplus c^{+}, \\ a^{+} \oplus b^{+} \oplus c^{-}, a^{+} \oplus b^{-} \oplus c^{+}, a^{-} \oplus b^{+} \oplus c^{+}, \\ \xcancel{a^{-} \oplus b^{-} \oplus c^{+}}, \xcancel{a^{-} \oplus b^{+} \oplus c^{-}}, \xcancel{a^{+} \oplus b^{-} \oplus c^{-}}, \\ \xcancel{a^{-} \oplus b^{-} \oplus c^{-}} \end{array} \right\}$$

- Together with existential raising (*ER*) and distributivity (Δ), delivers *at least* truth-conditions:

(14) Two boys sneezed.

$ER(\textbf{two} \cap [\![\text{boys}]\!])(\Delta(\textbf{sneezed}))$

$\Rightarrow \Delta(\textbf{sneezed})(a^+ \oplus b^+ \oplus c^+)$

$\vee \Delta(\textbf{sneezed})(a^+ \oplus b^+ \oplus c^-)$

$\vee \Delta(\textbf{sneezed})(a^+ \oplus b^- \oplus c^+)$

$\vee \Delta(\textbf{sneezed})(a^- \oplus b^+ \oplus c^+)$

- This strategy generalizes to complex numeral expressions, which can all be treated as predicates of pluralities:

(15) **exactly 2** $:= \{X \in D^{\pm} \mid \#X^{+} = 2\}$

(16) **between 3 and 5** $:= \{X \in D^{\pm} \mid 3 \leq \#X^{+} \leq 5\}$

(17) **less than 3** $:= \{X \in D^{\pm} \mid \#X^{+} < 3\}$

- Incorporating negative individuals immediately improves over a classical treatment of numerals as predicates with *ER* in some important respects:
  - Avoids van Benthem's problem with distributive predicates.
  - Avoids unwanted existential entailments for *less than n*
  - Allows "zero" to be treated as a numeral.

- In a classical setting, existential quantification renders upper-bounds inert; the following are equivalent (van Benthem 1986).
  - $\exists X[\#X = 2, X \in {}^*\mathbf{boy}, \forall x \in X, P(x)]$
  - $\exists X[\#X \geq 2, X \in {}^*\mathbf{boy}, \forall x \in X, P(x)]$
- Thanks to maximality in NP-extensions, this problem doesn't arise:

(18) $\mathbf{exactly\ 2} \cap [\![\text{boys}]\!] =$

$$\left\{ \begin{array}{c} \cancel{a^+ \oplus b^+ \oplus c^+}, \\ a^+ \oplus b^+ \oplus c^-, a^+ \oplus b^- \oplus c^+, a^- \oplus b^+ \oplus c^+, \\ \cancel{a^- \oplus b^- \oplus c^+}, \cancel{a^- \oplus b^+ \oplus c^-}, \cancel{a^+ \oplus b^- \oplus c^-}, \\ \cancel{a^- \oplus b^- \oplus c^-} \end{array} \right\}$$

- *ER* derives the attested truth-conditions; in my Sinn und Bedeutung poster, I applied this to the problem of cumulative readings (Brasoveanu 2013).

- In a classical setting, the predicative treatment of "less than *n*" leads to unwanted existential entailments (Buccola & Spector 2016).
  - $\exists X[\#X < n, X \in {}^{*}\mathbf{boy}(X), P(X)]$
- This is because there are no pluralities with cardinality 0; the minimal pluralities are atoms.
- This problem doesn't arise here, thanks to wholly negative pluralities.

(19) **less than 2** $\cap$ $[\![\text{boys}]\!] =$

$$\left\{\begin{array}{c} \cancel{a^{+} \oplus b^{+} \oplus c^{+}}, \\ \cancel{a^{+} \oplus b^{+} \oplus c^{-}}, \cancel{a^{+} \oplus b^{-} \oplus c^{+}}, \cancel{a^{-} \oplus b^{+} \oplus c^{+}}, \\ a^{-} \oplus b^{-} \oplus c^{+}, a^{-} \oplus b^{+} \oplus c^{-}, a^{+} \oplus b^{-} \oplus c^{-}, \\ a^{-} \oplus b^{-} \oplus c^{-} \end{array}\right\}$$

- In a classical setting, a predicative treatment of "zero NPs" isn't viable; since the minimal pluralities are atoms, a predicative treatment of "zero" leads to a necessary contradiction.
- A treatment of "zero" is straightforward here, with the proviso that it must have an *exactly* semantics to avoid a necessary tautology (Bylinina & Nouwen 2018).

(20) $\textbf{zero} = \{X \in D\pm \mid \#X = 0\}$

(21) $\textbf{zero} \cap [\![\text{boys}]\!] =$

$$\left\{ \begin{array}{c} \cancel{a^+ \oplus b^+ \oplus c^+}, \\ \cancel{a^+ \oplus b^+ \oplus c^-}, \cancel{a^+ \oplus b^- \oplus c^+}, \cancel{a^- \oplus b^+ \oplus c^+}, \\ \cancel{a^- \oplus b^- \oplus c^+}, \cancel{a^- \oplus b^+ \oplus c^-}, \cancel{a^+ \oplus b^- \oplus c^-}, \\ a^- \oplus b^- \oplus c^- \end{array} \right\}$$

On the bottom element

- Buccola & Spector (2016) entertain extending Link's plural ontology with a *bottom element* $\bot$, s.t., $\#\bot = 0$, in order to solve the existential entailment problem with *less than n*
- Bylinina & Nouwen (2018) consider the same move, in order to give a principled semantics for "zero".
- In the current setting, *maximal, wholly negative pluralities* play the same role as the bottom element.
  - This however was not tailored as a solution for these problems, but falls out as a happy accident.
  - Ask me about presupposition projection for an independent argument that negative individuals are preferable to the bottom element.

- Tellingly, none of the problems I've noted arise on a GQ-theoretic treatment of numerals either, since GQ-theory makes no reference to pluralities:

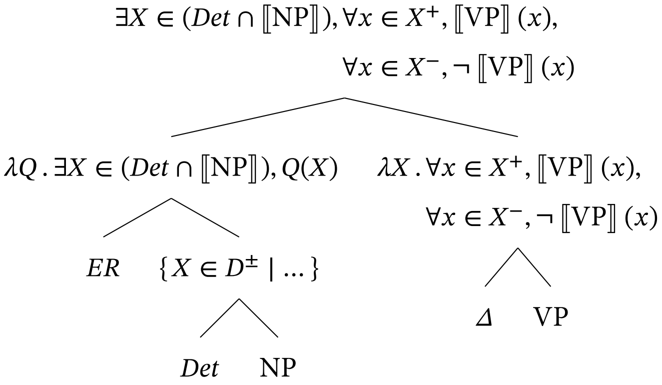
(22) $\textbf{less than 3}(R, S) \iff \#(R \cap S) < 3$

- (22) of course holds if $R \cap S$ is empty.
- Negative individuals allow us to retain *both* the expressive advantages of GQ-theory, and the advantages of treating numerals as predicates of pluralities.

- In the following section, I'll demonstrate that negative individuals are not just handy for numeral semantics.
- Not just numerals, but *all conservative determiners* may be defined as predicates of pluralities.
- The LF for quantificational statements generalizes the compositional strategy developed for numerals.
- Furthermore, non-conservative determiners are *not expressible* as predicates of pluralities; if all determiners are predicates, the conservativity universal is explained.

Determiners and conservativity

A unified LF for quantificational statements



- We've already seen that with negative individuals, we can easily define both bare and complex numerals as predicates of pluralities.
- This strategy can easily be extended to existential/universal determiners, by placing constraints on X^+ and X^-.

(23) **some** $= \{X \in D^\pm \mid X^+ \neq \varnothing\}$

(24) **all** $= \{X \in D^\pm \mid X^- = \varnothing\}$

(25) **no** $= \{X \in D^\pm \mid X^+ = \varnothing\}$

(26) **not all** $= \{X \in D^\pm \mid X^- \neq \varnothing\}$

- It can easily be verified that these entries give rise to the right truth-conditions.
- In particular, there is always a unique maximal NP plurality in $D^{\pm}$ with no negative parts, and a unique maximal NP plurality in $D^{\pm}$ with no positive parts.

(27) All boys sneeze.
$\Rightarrow ER(\textbf{all} \cap [\![\text{boys}]\!])(\Delta([\![\text{sneeze}]\!])) \Rightarrow \Delta([\![\text{sneeze}]\!])(a^{+} \oplus b^{+} \oplus c^{+})$

(28) No boys sneeze.
$\Rightarrow ER(\textbf{no} \cap [\![\text{boys}]\!])(\Delta([\![\text{sneeze}]\!])) \Rightarrow \Delta([\![\text{sneeze}]\!])(a^{-} \oplus b^{-} \oplus c^{-})$

- This strategy extends to proportional determiners via cardinality comparisons.

(29) $\textbf{most} = \{ X \in D^{\pm} \mid \#X^{+} > \#X^{-} \}$

(30) $\textbf{exactly half} = \{ X \in D^{\pm} \mid \#X^{+} = \#X^{-} \}$

(31) $\textbf{most} \cap [\![\text{boys}]\!] =$

$$\left\{ \begin{array}{c} a^{+} \oplus b^{+} \oplus c^{+}, \\ a^{+} \oplus b^{+} \oplus c^{-}, a^{+} \oplus b^{-} \oplus c^{+}, a^{-} \oplus b^{+} \oplus c^{+}, \\ \xcancel{a^{-} \oplus b^{-} \oplus c^{+}}, \xcancel{a^{-} \oplus b^{+} \oplus c^{-}}, \xcancel{a^{+} \oplus b^{-} \oplus c^{-}}, \\ \xcancel{a^{-} \oplus b^{-} \oplus c^{-}} \end{array} \right\}$$

- What would it take to define a non-conservative determiner in this system?
  - Take the Härtig quantifier I:

(32) $I(A, B) \iff \#A = \#B$

- In the current system, a *Det* is a predicate that composes with a plural NP via intersective modification. Therefore:

$$(Det \cap [\![\text{NP}]\!]) \subseteq [\![\text{NP}]\!]$$

- The NP itself delimits possible determiner meanings; each plurality $X \in [\![\text{NP}]\!]$ encodes information, for each $x \in A$, about whether x is true or false of B.
  - See (Westerståhl 2024) for a related notion of restricted quantification.

- In order to define I, we need to access just the scope set B independently of the restrictor A.
- It's clearly not possible to access B by taking a subset of $[\![\text{NP}]\!]$:
    - Given a maximal NP plurality X:
        - $X^+ \cup X^- = A$
        - $X^+ \cap X^- = \varnothing$
        - $X^+ = A \cap B$
        - $X^- = A - B$
- A standard conceptualization of conservativity is that it rules out determiner meanings which make reference to the scope, not relative to the restrictor.

- In a sentence of the form [*Det* NP VP], each element of ⟦NP⟧ corresponds to a *complete answer* to the question, "who of NP did VP?".
- Selecting a subset of ⟦NP⟧ will invariably deliver a proposition that is relevant (in the sense of von Fintel & Heim 2023), relative to the partition induced by "who of A did B?".
- Conjecture: conservative, but not non-conservative determiners make $Det(A, B)$ relevant to "who of A did B?".

- Let R be an arbitrary restrictor.
- Consider $\mathbf{Max}\{X \in D^{\pm} \mid \forall x \in X^{+} \cup X^{-}, R(x)\}$.
- As we've seen, this set is isomorphic to the set of functions $\mathbb{R} := \{f \mid f : R \mapsto \{1, 0\}\}$
  - Assuming $R := \{a, b\}$

$$\mathbb{R} = \left[\overbrace{\begin{bmatrix} a \to 1 \\ b \to 1 \end{bmatrix}}^{a^{+} \oplus b^{+}}, \overbrace{\begin{bmatrix} a \to 1 \\ b \to 0 \end{bmatrix}}^{a^{+} \oplus b^{-}}, \overbrace{\begin{bmatrix} a \to 0 \\ b \to 1 \end{bmatrix}}^{a^{-} \oplus b^{+}}, \overbrace{\begin{bmatrix} a \to 0 \\ b \to 0 \end{bmatrix}}^{a^{-} \oplus b^{-}} \right]$$

- $Det(\mathbb{R}) \subseteq \mathbb{R}$ (determiners are restrictive modifiers).
- For example, "most" picks out the smallest subset of $\mathbb{R}$ containing every function that maps more elements of R to 1 than 0.
  - $f^{+} = \{\, x \in \mathbf{dom}(f) \mid f(x) = 1 \,\}$
  - $f^{-} = \{\, x \in \mathbf{dom}(f) \mid f(x) = 0 \,\}$

(33) $\textbf{most boys} \approx \{\, f \mid f : \textbf{boy} \mapsto \{\, 1, 0 \,\}, f^{+} > f^{-} \,\}$

$$\left\{ \begin{bmatrix} a \to 1 \\ b \to 1 \\ c \to 1 \end{bmatrix}, \begin{bmatrix} a \to 1 \\ b \to 1 \\ c \to 0 \end{bmatrix}, \begin{bmatrix} a \to 1 \\ b \to 0 \\ c \to 1 \end{bmatrix}, \begin{bmatrix} a \to 0 \\ b \to 1 \\ c \to 1 \end{bmatrix} \right\}$$

Non-conservative determiners are not-expressible

- How does $Det(\mathbb{R})$ combine with the scope $S : D \mapsto \{1, 0\}$.
- $ER + \Delta$ leads to the requirement there is an $f \in Det(\mathbb{R})$, s.t., f and S agree on $\mathbf{Dom}(f)$.
- The resulting truth-conditions of a quantificational statement can be reformulated in terms of Boolean functions:

$$\exists f \in Det(\mathbb{R}), \forall x \in \mathbf{dom}(f)(f(x) \iff S(x))$$

Informal demonstration cont.

$$\exists f \in Det(\mathbb{R}), \forall x \in \mathbf{dom}(f)(f(x) \iff S(x))$$

- It's obvious from this formulation that $S - R$ cannot effect the resulting truth-conditions, since as long as $Det(\mathbb{R}) \subseteq \mathbb{R}$, any choice of f is s.t., $\mathbf{dom}(f) = R$
- To determine whether f and S agree on $\mathbf{Dom}(f)$, we only need to look at $\mathbf{Dom}(f) \cap S$, i.e., $R \cap S$.
  - Any determiner expressible in this way must be conservative.

Any conservative determiner is expressible

- Let R_{Cons} be a conservative determiner.

$$R_{Cons}(A, B) \iff R_{Cons}(A, A \cap B)$$

- R_{Cons} gives rise to a set of boolean functions as follows:
  - $f^+ \cup f^- \approx (A \cap B) \cup (A - B) \approx A$
  - $f^+ \approx A \cap B$

(34) $\{ f \mid \exists X \in D, f : X \mapsto \{1, 0\}, R_{Cons}(f^+ \cup f^-, f^+) \}$

- This is isomorphic to a subset of $D^{\pm}$.

(35) $$\left\{ f \;\middle|\; \begin{array}{l} \exists X \in D, f : X \mapsto \{1,0\}, \\ \mathbf{most}(f^+ \cup f^-, f^+) \end{array} \right\}$$

(36) $$\left\{ f \;\middle|\; \begin{array}{l} \exists X \in D, f : X \mapsto \{1,0\}, \\ \#((f^+ \cup f^-) \cap f^+) > \#((f^+ \cup f^-) - f^+) \end{array} \right\}$$

(37) $$\equiv \left\{ f \;\middle|\; \begin{array}{l} \exists X \in D, f : X \mapsto \{1,0\}, \\ \#f^+ > \#f^- \end{array} \right\}$$

(38) $$\equiv \left\{ f \;\middle|\; \begin{array}{l} \exists X \in D, f : X \mapsto \{1,0\}, \\ \#f^+ > \#f^- \end{array} \right\} \cap \{ f \mid f : \mathbf{boy} \to \{1,0\}\}$$

(39) $$\{ f \mid f : \mathbf{boy} \to \{1,0\}, \#f^+ > \#f^- \}$$

(40) $$\exists f : \mathbf{boy} \mapsto \{1,0\}, \#f^+ > \#f^-, \\ \forall x \in \mathbf{boy}[f(x) \iff \mathbf{sneeze}(x)]$$

Extensions and open issues

- The current framework struggles to account for the distinction between:
  - "No boy" vs. "no boys"
  - "Some boy" vs. "some boys"
  - "Every boy" vs. "all boys"
- No worse than GQ-theory, but it order to give a uniform semantics for determiners, we need a more sophisticated notion of plurality/singularity.

- Relatedly, how to account for collective predication?

  (41) Some boys met in the park.

  (42) #Some boy met in the park.

- A natural move is to also consider positive/negative counterparts of i-sums, e.g., $(a \oplus b \oplus c)^{-}$ (Justin Bledin, p.c.).
  - singular NPs range over maximal sums of *atomic* counterparts; plural NPs range over maximal sums of *plural* counterparts.
  - Exploring the ramifications of this set-up, and its applications to semantic singularity/plurality and collective predication is the next step in this research program.

- I've developed a system for determiner meanings which allows us to make sense of Logical Forms that look like the following:

(43) Most boys sneezed.
There exists an X s.t., **most**(X) $\wedge$ **boys**(X) $\wedge$ **sneezed**(X).

- I've suggested that this solves the expressivity problem that arises with determiners *qua* higher-order functions.
  - If determiners are uniformly predicates of pluralities, all (attested) conservative determiners can be expressed, but non-conservative determiners can't be expressed.
- An explicit comparison with the structural approach to conservativity (Romoli 2015) is left for another occasion.

$\mathcal{F}in$

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